Department of Economics and Business
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Ranking paths in stochastic time-dependent networks

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VeRoLog - June 18-20, 2012

Agenda

Stochastic time-dependent networks

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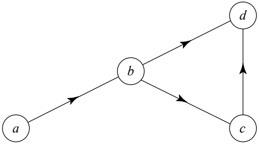
- No waiting allowed at nodes (arrival = departure).

A hypergraph representation

$(u,v),t$	$(a,b),0$	$(b,c),1$	$(b,c),2$	$(b,d),1$	$(b,d),2$	$(c,d),2$	$(c,d),3$
$I(u,v,t)$	$\{1,2\}$	$\{2,3\}$	$\{3\}$	$\{3\}$	$\{6\}$	$\{3,4\}$	$\{4,5\}$
p_{ijt}	$\{\frac{1}{2},\frac{1}{2}\}$	$\{\frac{1}{2},\frac{1}{2}\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{\frac{1}{2},\frac{1}{2}\}$	$\{\frac{1}{2},\frac{1}{2}\}$



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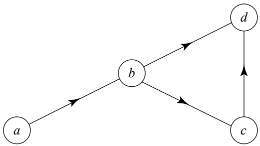

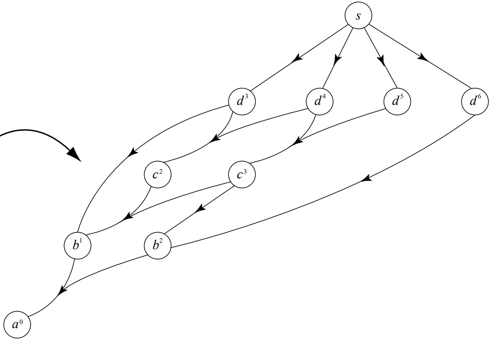

Route choice in STD networks

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In this talk we consider a priori routing

Route selection criteria

We consider the following objectives:

- Minimizing the expected travel time (MET)

Under MEC we define the following costs:

- $c(u, v, t)$ – cost of leaving node u at time t along arc (u, v).
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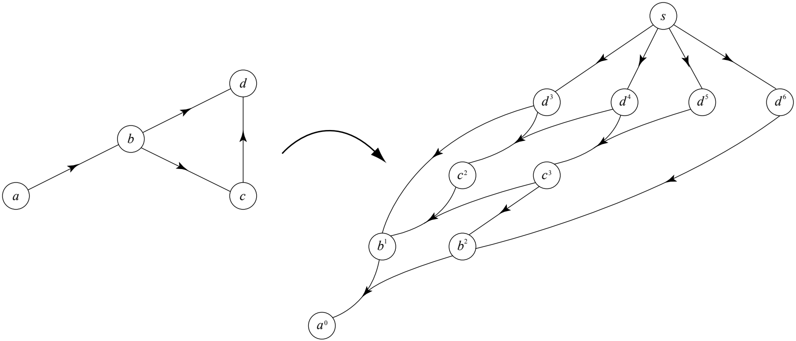
Known results

- Time-adaptive route choice – Finding the best strategy $\Leftrightarrow$ finding a minimum weight hyperpath.
- A priori route choice – Finding the shortest path is NP-hard.

Example MEC (time-adaptive routing)



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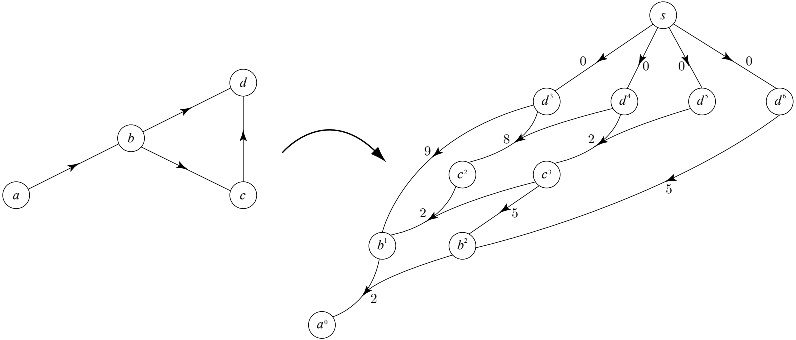

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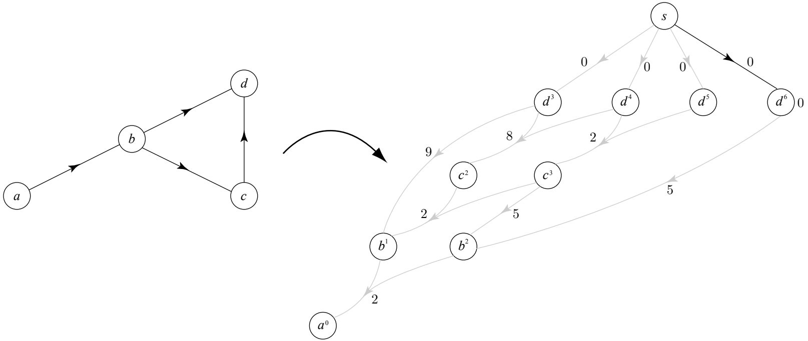

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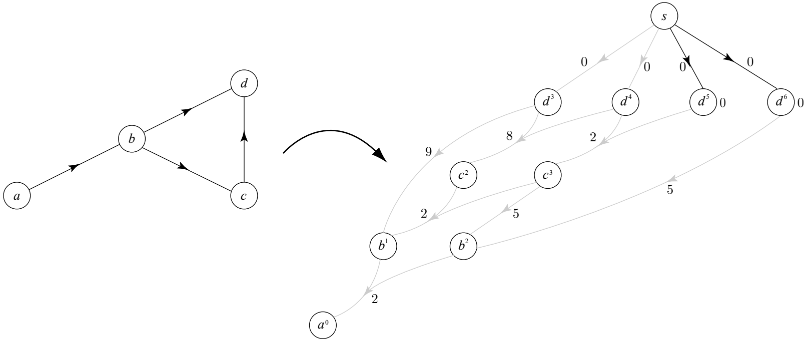

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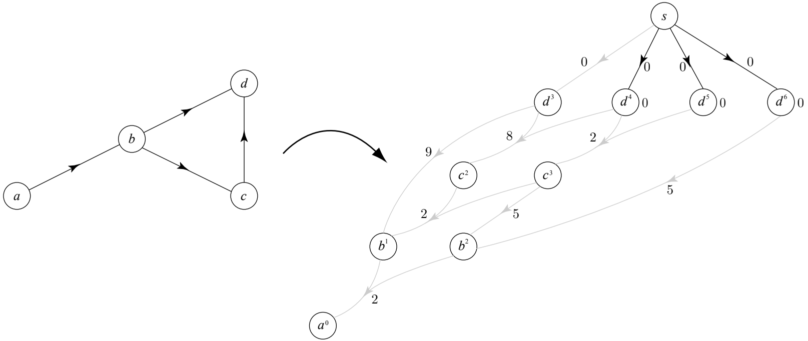

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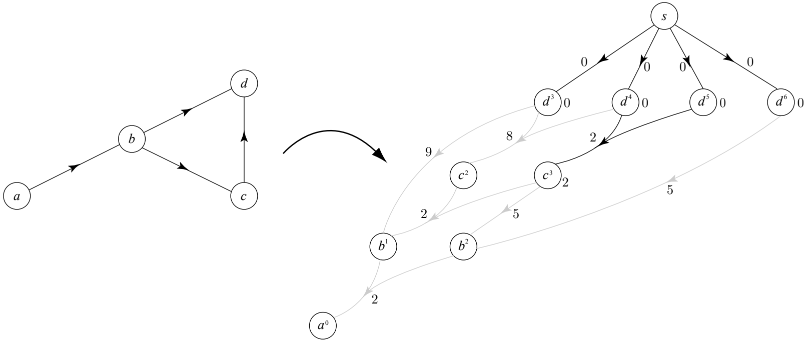

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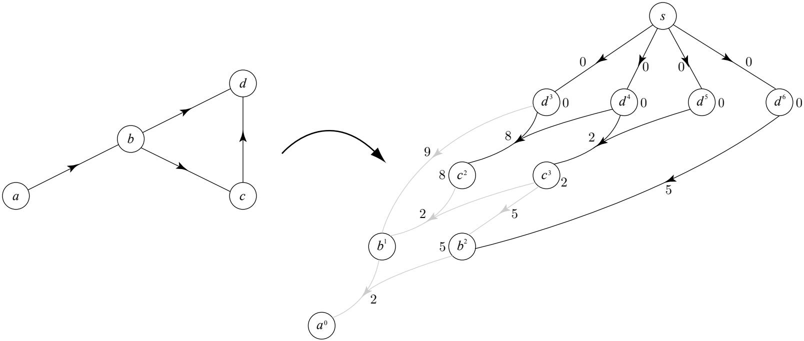

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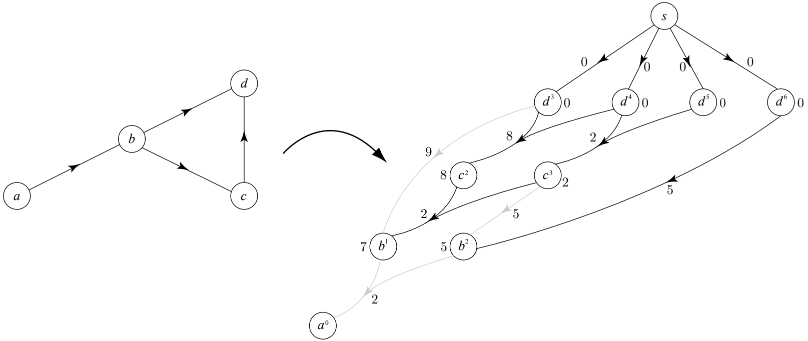

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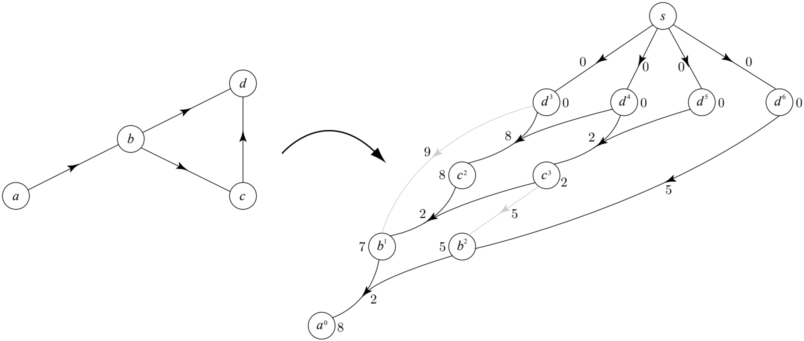

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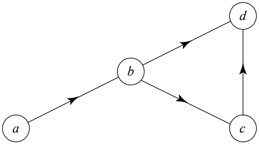

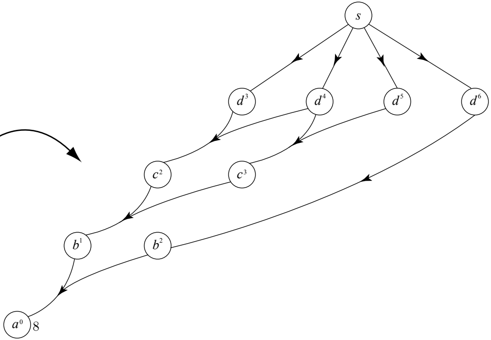

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Ranking scheme (deterministic case)

Let $\mathcal{P}$ denote the set of paths in G and p the shortest path.
Generic ranking scheme based on lower bounds:

1. Patition $\mathcal{P} \setminus \{p\}$ into q disjoint subsets $\mathcal{P}^1, \dots, \mathcal{P}^q$ based on p

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- Finding the best routing strategy gives us a lower bound on the expected cost of the shortest path
- The best routing strategy may be used to partition the solution space

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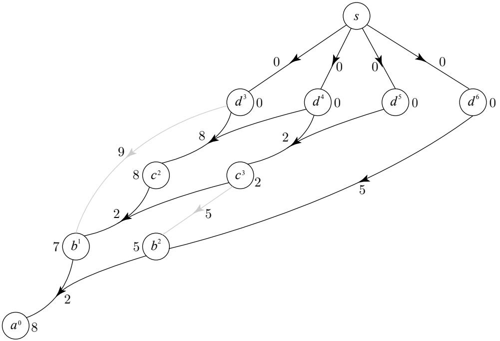
Optimal routing strategy



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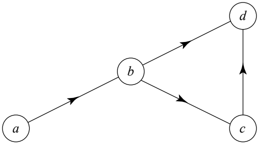
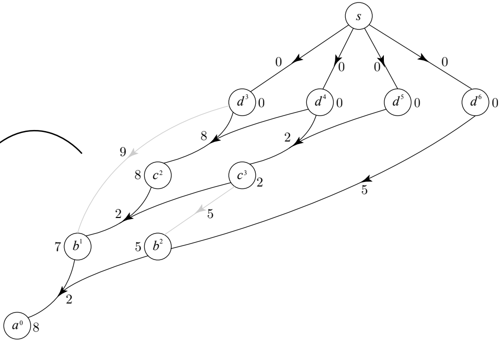
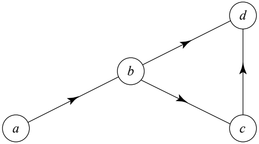




Choosing subpath



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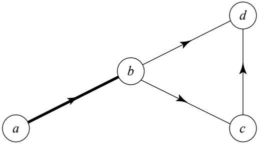

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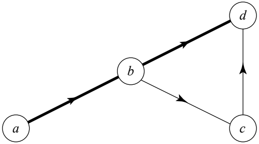

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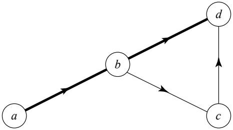

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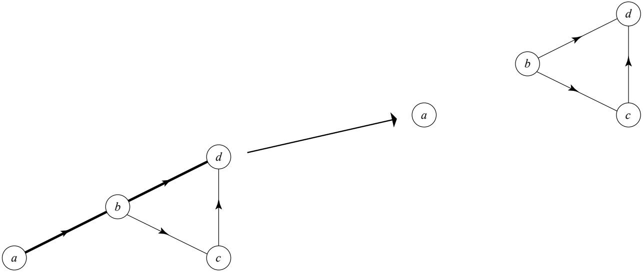

$\mathcal{P}^1$

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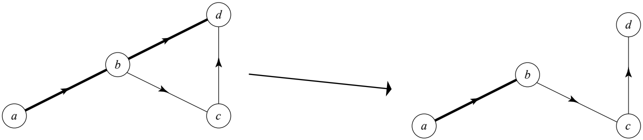

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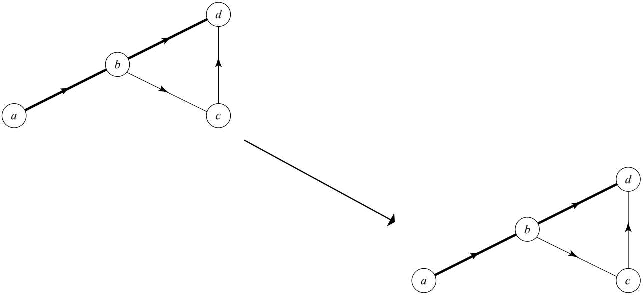

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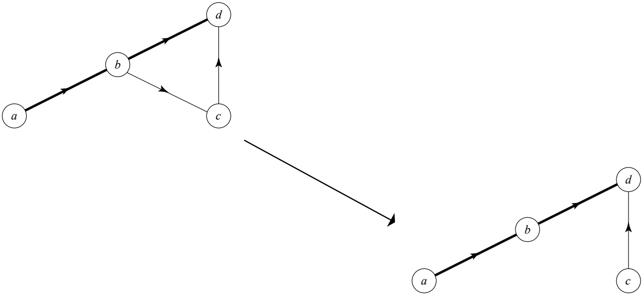

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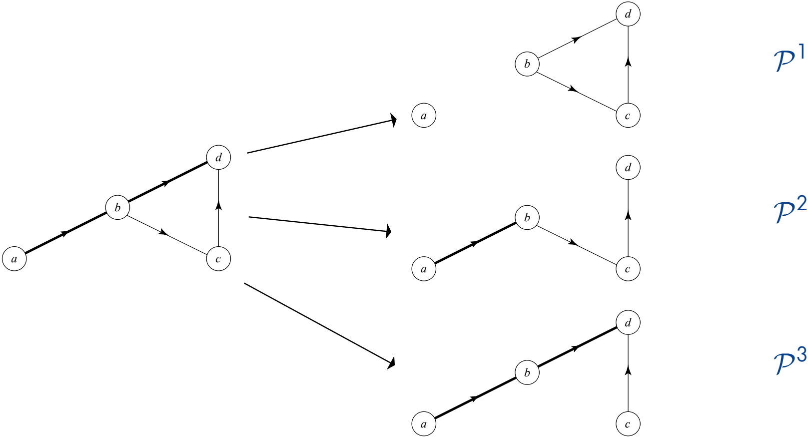

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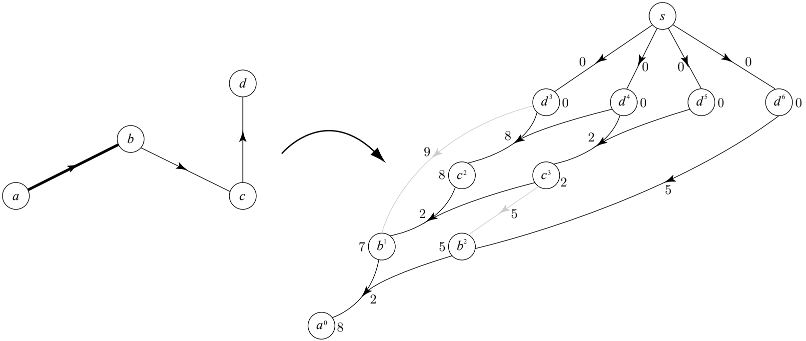

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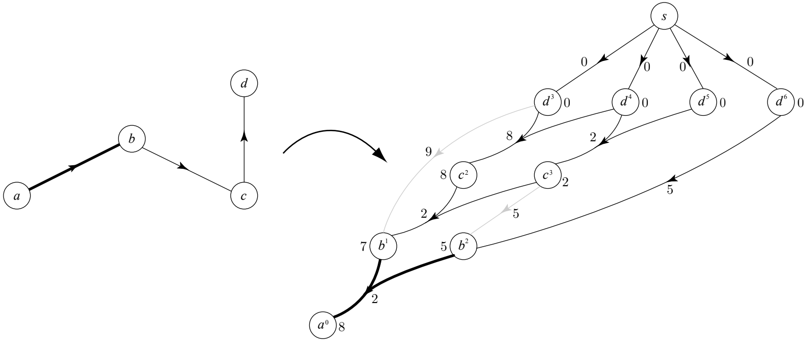

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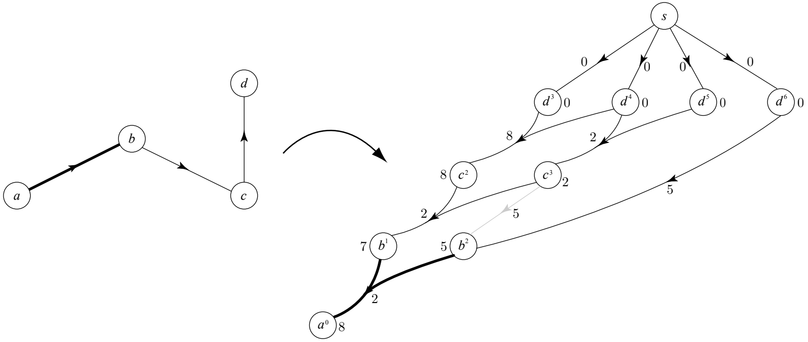

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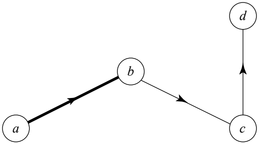

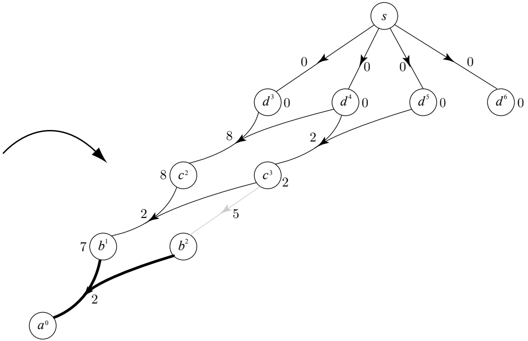

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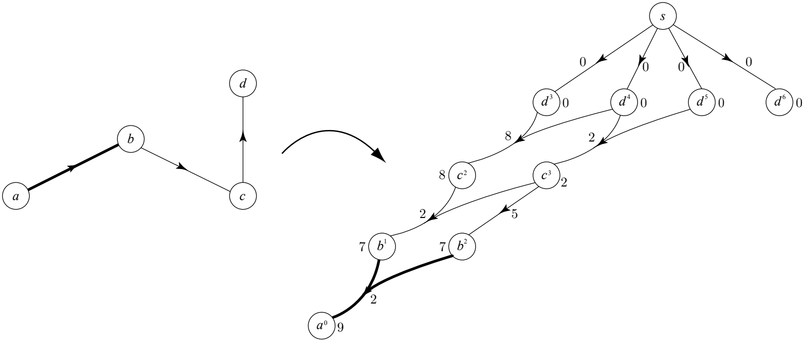

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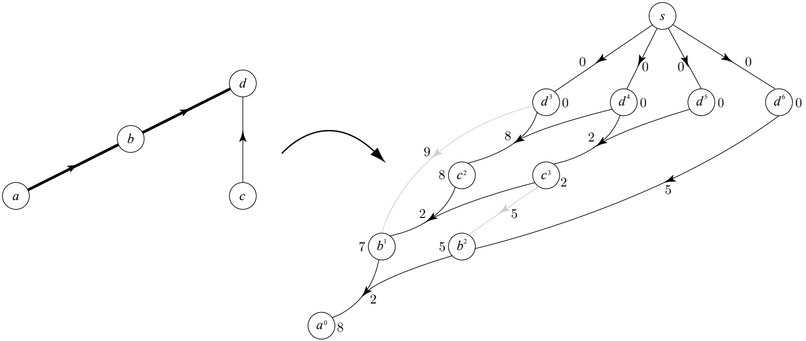

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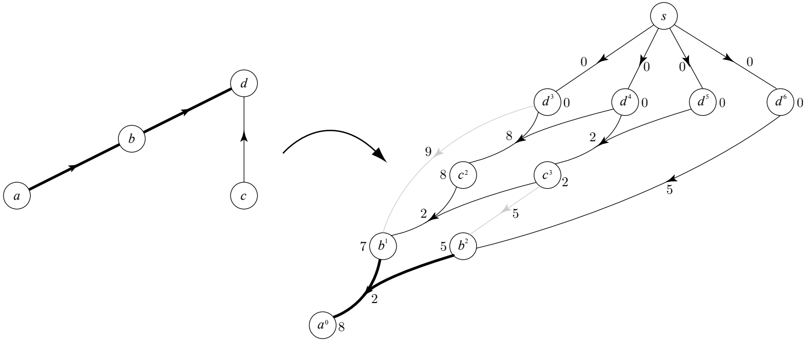

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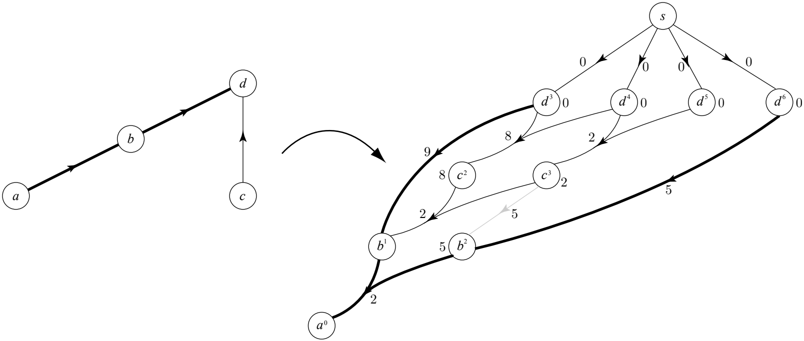

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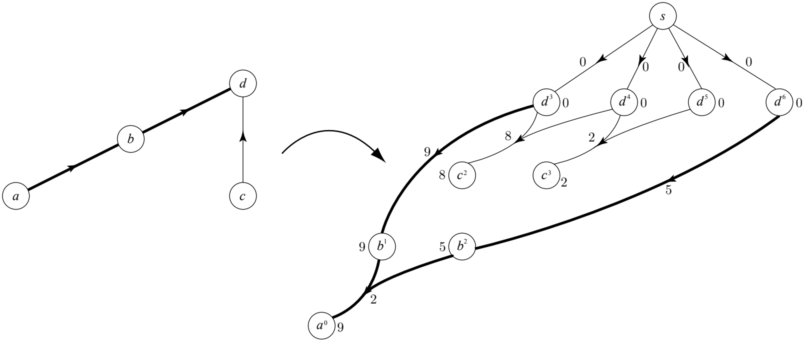

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Generator

- TEGP generator used (`www.research.relund.dk`).

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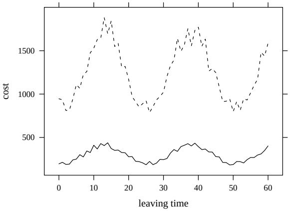
- Time unit 5 min, off-peak mean travel time [2, 6] (100% increase in peaks).
- Off-peak costs [1, 1000] (100% increase in peaks) + random noise (10%).



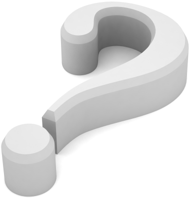
Test instances

	peak dependent costs			random costs		
Class	1	2	3	4	5	6
Grid size	5×10	10×10	20×10	5×10	10×10	20×10
n	2320	7573	21454	1497	3961	11856
m	7809	27278	79570	5056	14295	43991
H	118	156	237	75	101	155

Results ($K = 100$)

Class	ite_K	CPU_K	ite_1	CPU_1	$\delta(u) = 1$	$\delta(u) = 2$	$\delta(u) = 3$	$\delta(u) = 4$	inc	$inc_{S\text{-}PS}$
1	356	8	13	0.3	82	14	4	1	49	5
2	454	37	11	1.0	82	14	4	0	17	7
3	2359	875	133	53.5	85	13	2	0	10	5
4	1427	25	133	2.5	50	25	20	5	18	43
5	9942	564	2722	157.9	39	21	27	12	8	54
6	203479	34227	89519	15127.5	25	16	27	31	3	59

Questions



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