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Optimal replacement policies for dairy cows based on daily yield measurements

Case example: Markov decision processes

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Dept. of Large Animal Sciences, University of Copenhagen, Denmark

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Biosens II

- Biosens II: Improved monitoring and management of dairy production based on on-farm biosensors
- Goal: Better detection of oestrus and illnesses
- Focus on biomarkers in milk (progesterone, LDH, yield, etc.)
- Commercial partner Lattec I/S (FOSS A/S and DeLaval AB)
- Five year project (2007-2011). Budget ≈5 mill EUR
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Project 2.3: Economic value of the dairy cow

Find optimal strategy for each cow w.r.t. replacement, treatment and reproduction (economic point of view).

- Many papers about the dairy cow replacement problem but limited use in pratice. Reasons could be:
  - Often large and complex models.
  - Parameters in the model must be estimated, i.e. data collection frameworks at herd level must exist.
  - Stage length: one month up to a year → no assistance when to inseminate, treat or cull the cow in the current month.
- Bio-sensors and cow specific traits/interventions exists in modern dairy herds → parameters can be estimated on a daily basis.

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Develop MDP with daily stages based on daily yield measurements.

Problem

- Assign an economic value to a dairy cow during lactation
- Calculate the optimal replacement strategy based on the economic value
- Assume daily yield measurements available

Problem, models and results

Background

Overview

→ Problem and model

MDP intro

Dairy HMDP Model

State space model

Embedding the SSM

HMDP results

Status/future work

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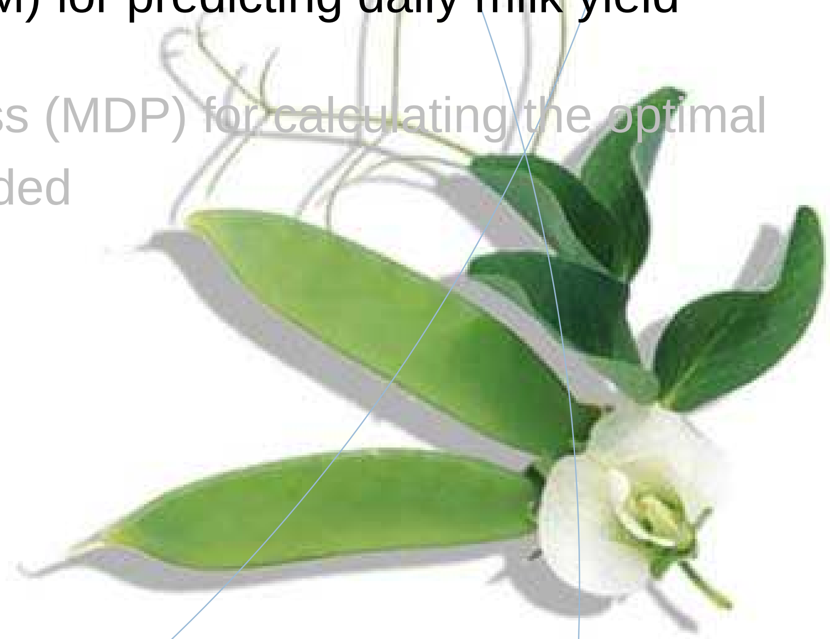
Models

- Use a state space model (SSM) for predicting daily milk yield
- Use a Markov decision process (MDP) for calculating the optimal strategy with the SSM embedded



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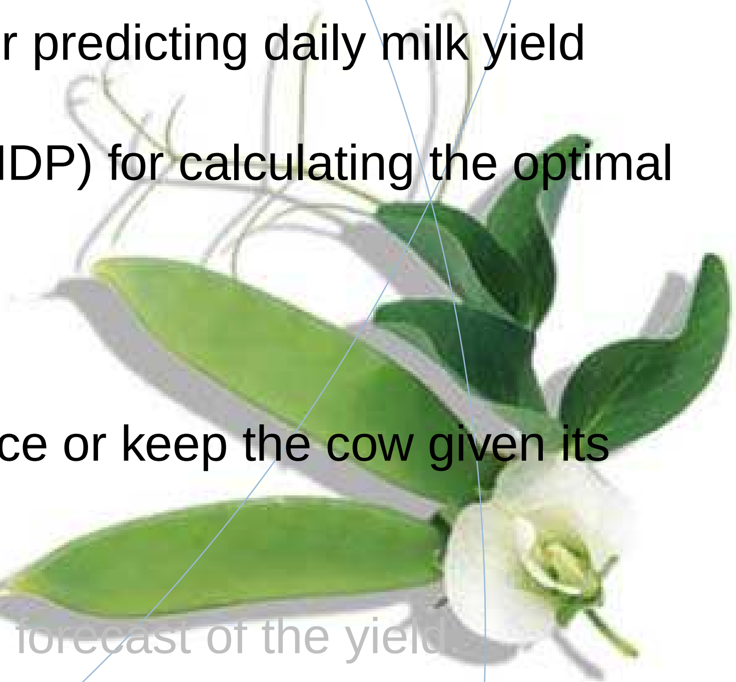
Results

- A strategy saying whether to replace or keep the cow given its current state
- An economic value of the cow and forecast of the yield



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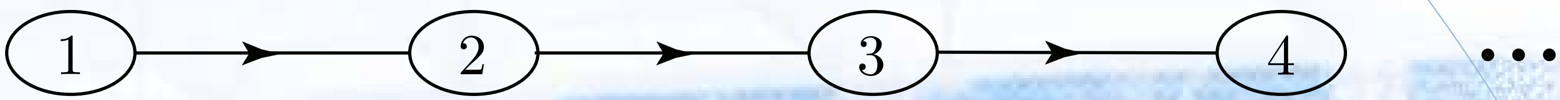
What is an MDP?



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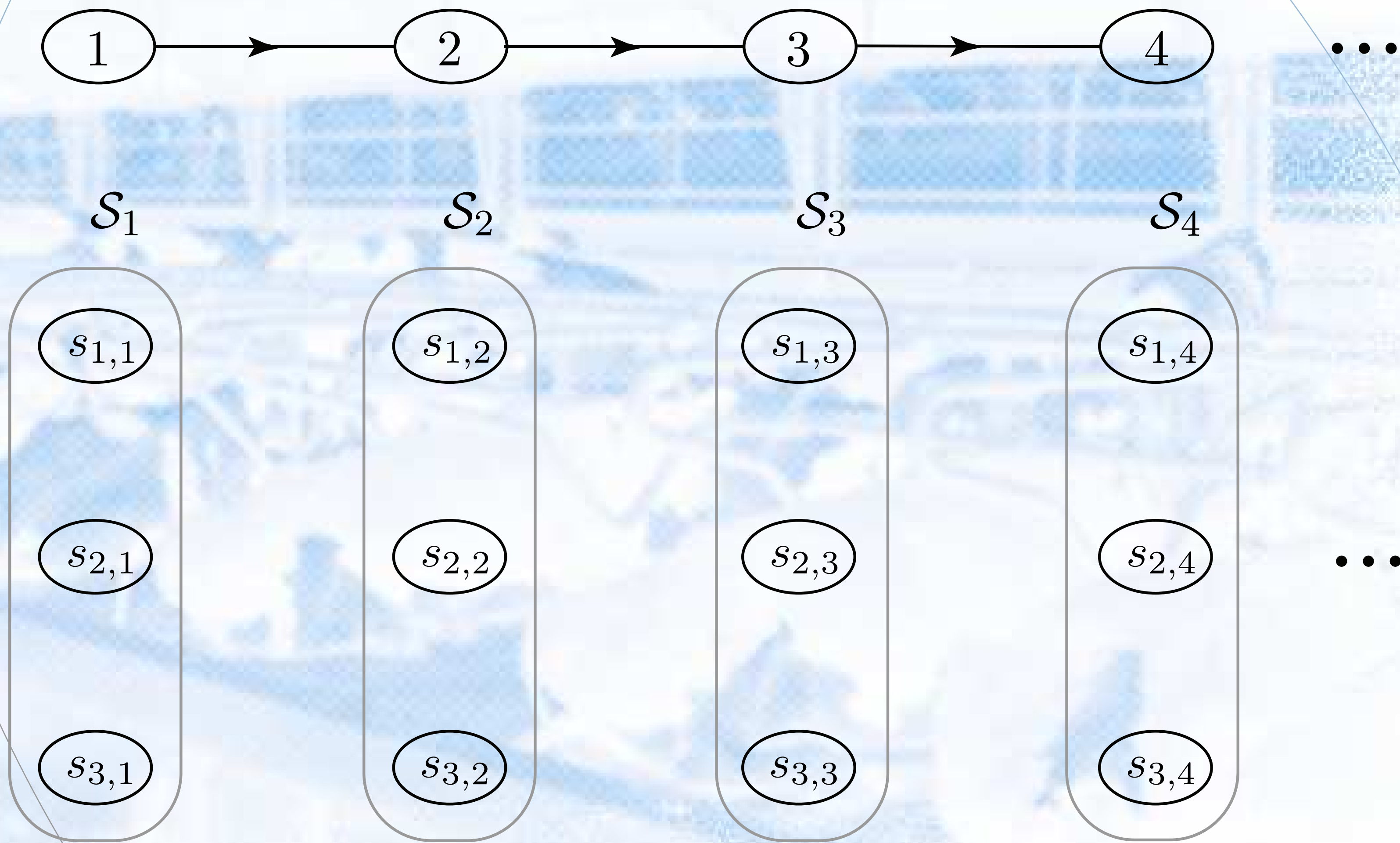

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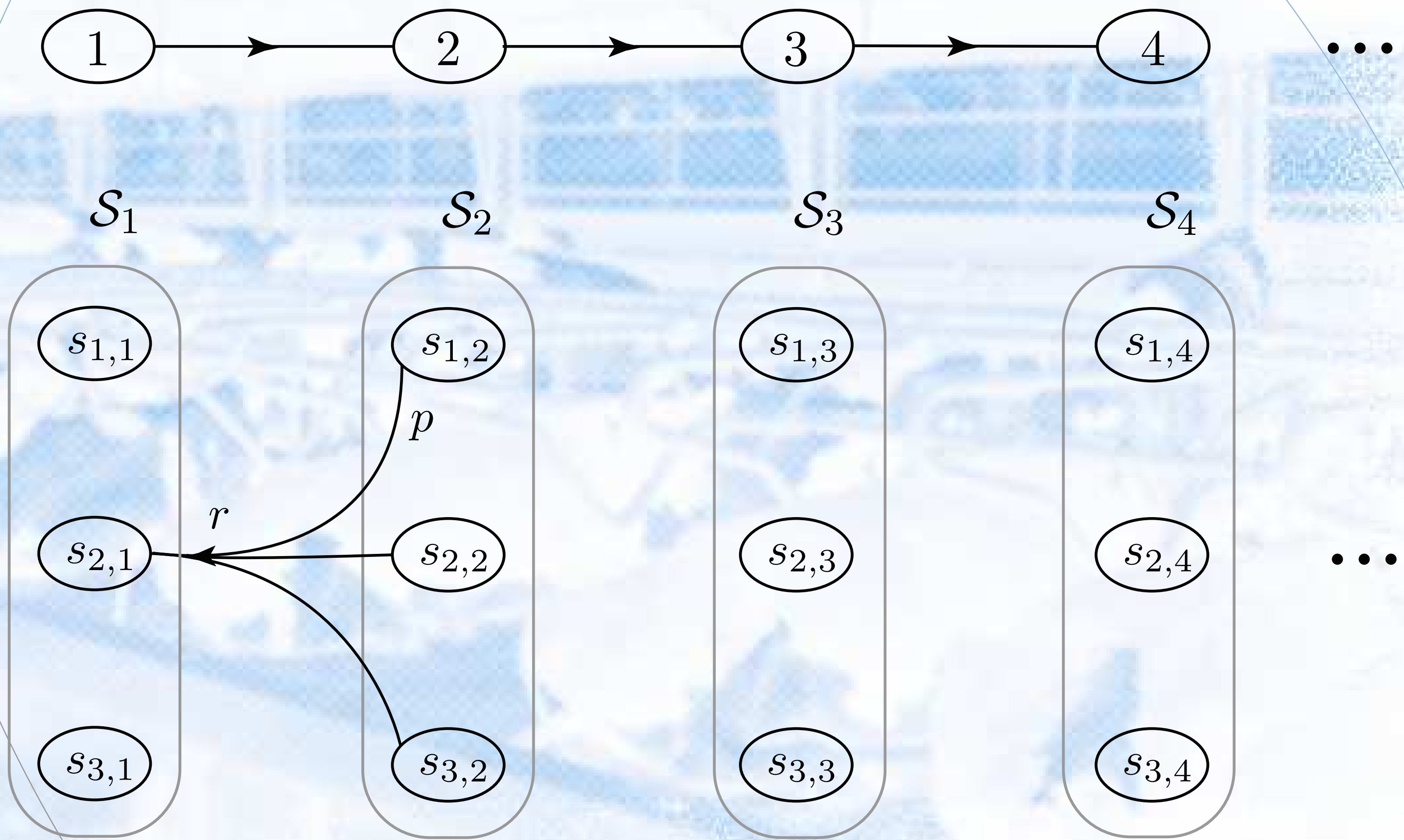

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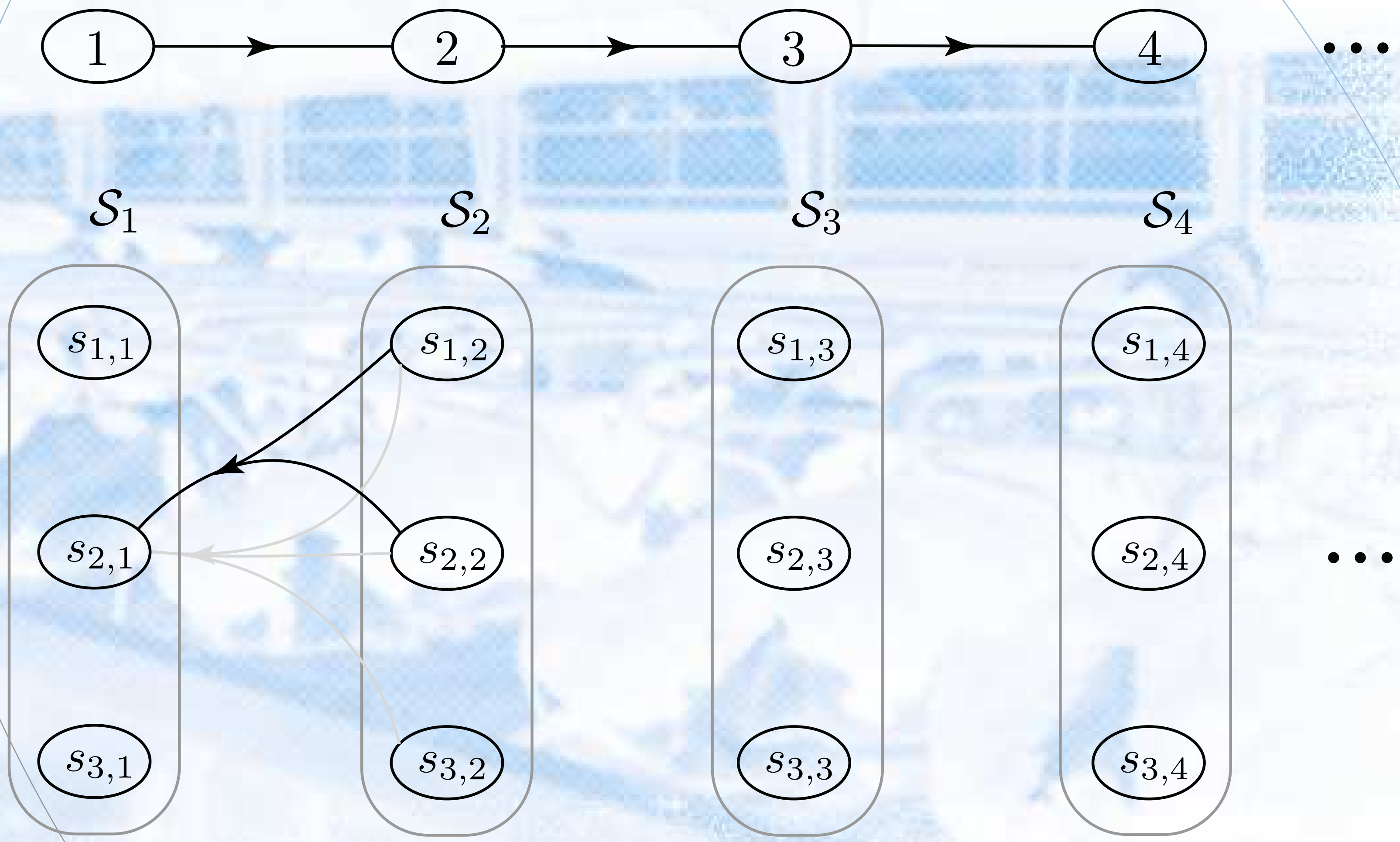

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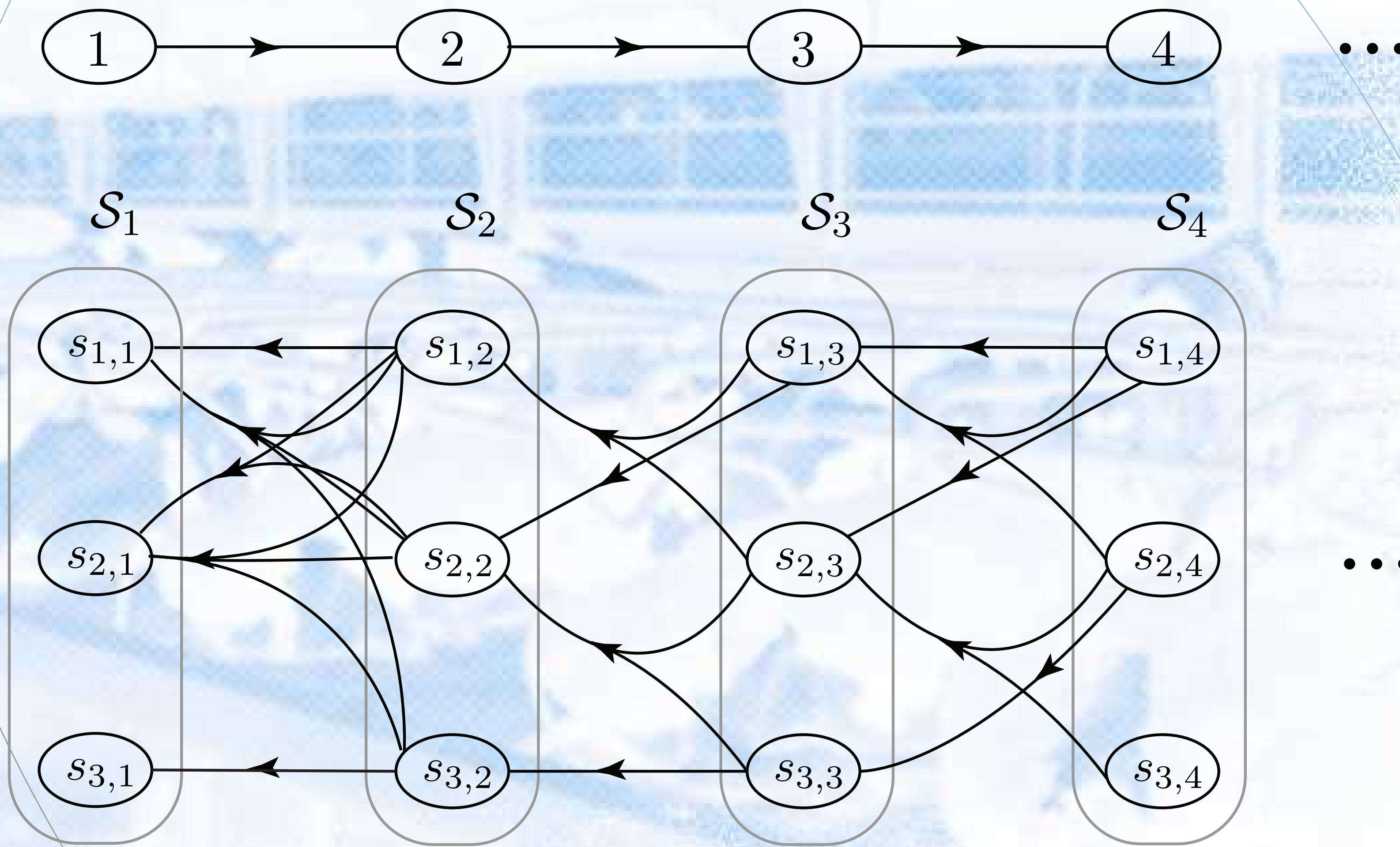

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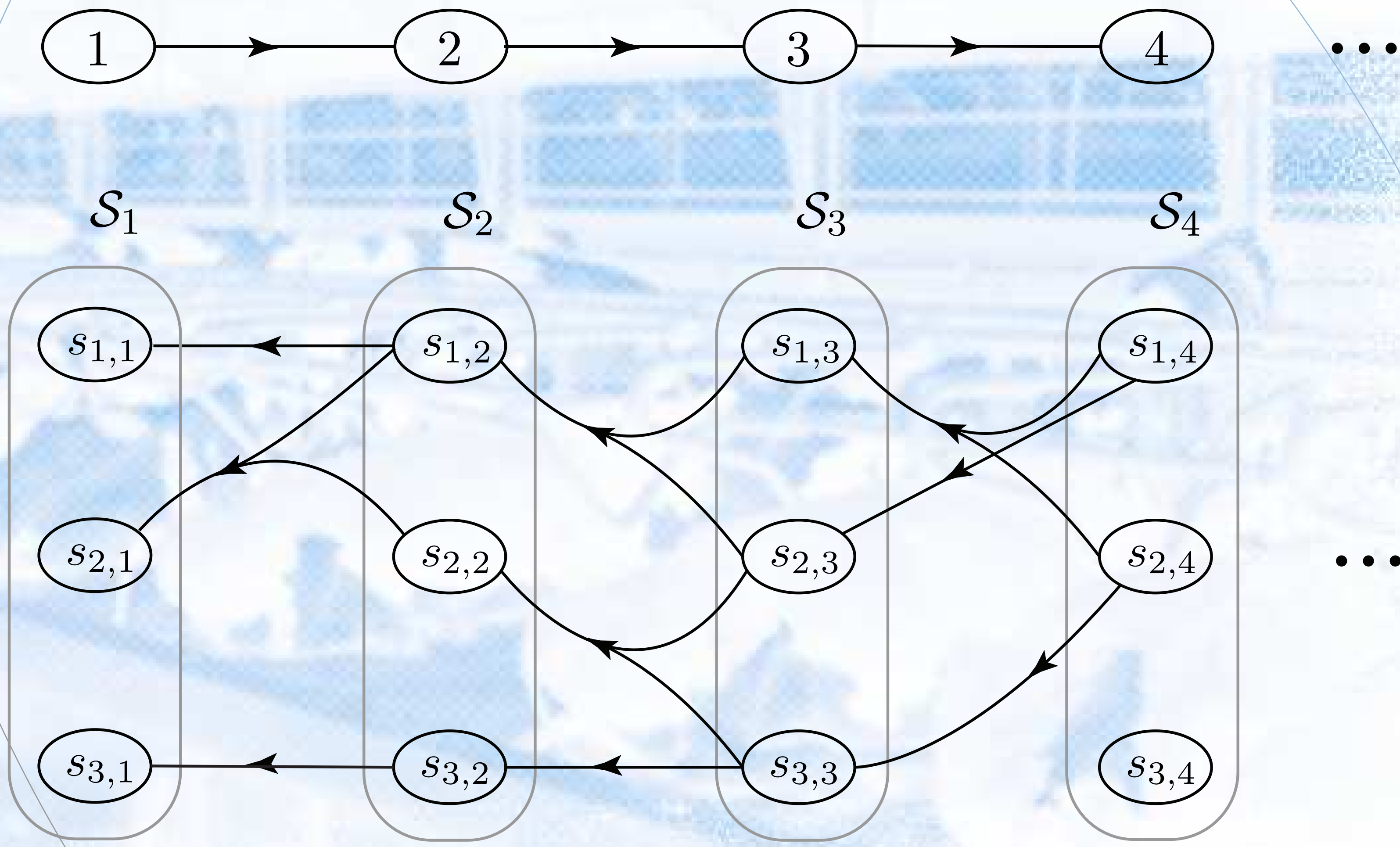
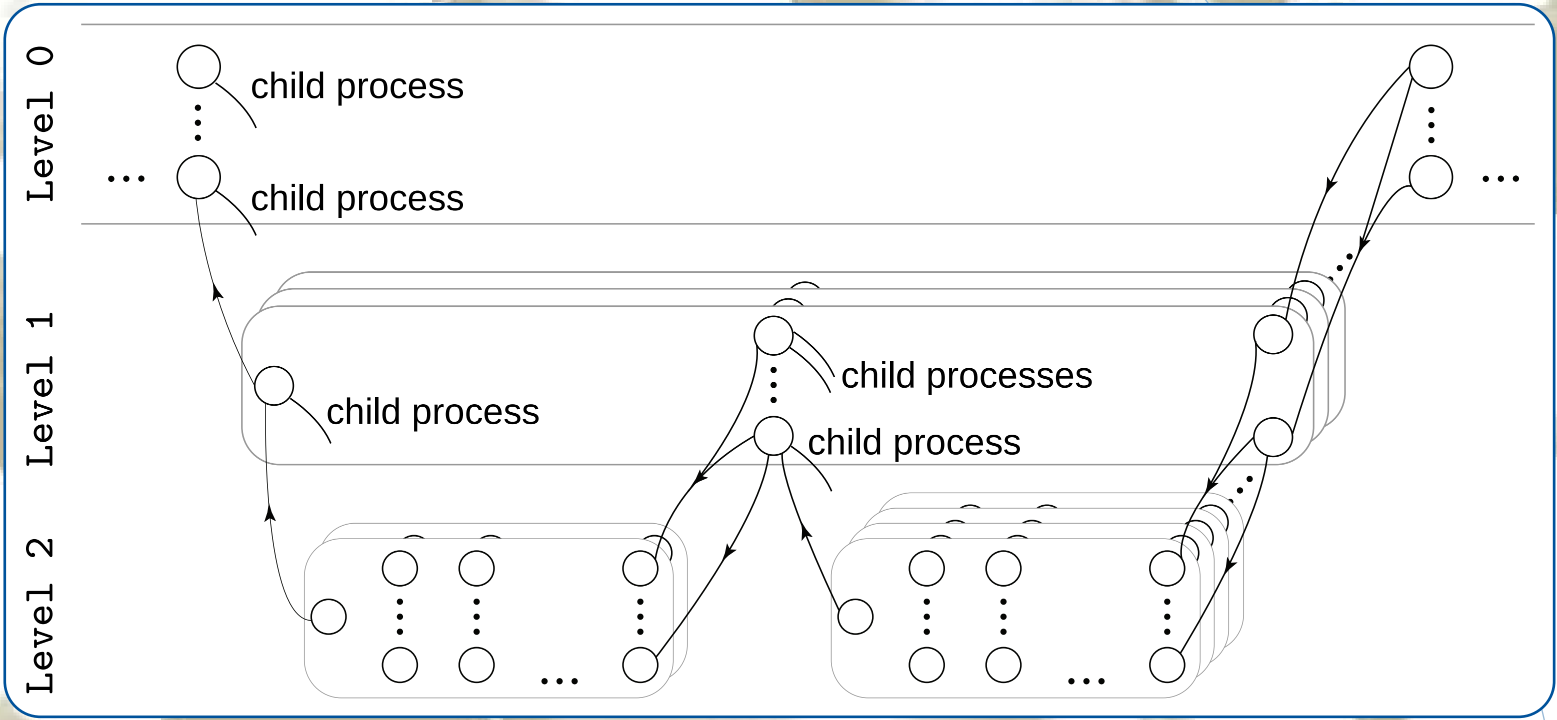


Hierarchical MDP (HMDP)



Lactation cycle of the cow

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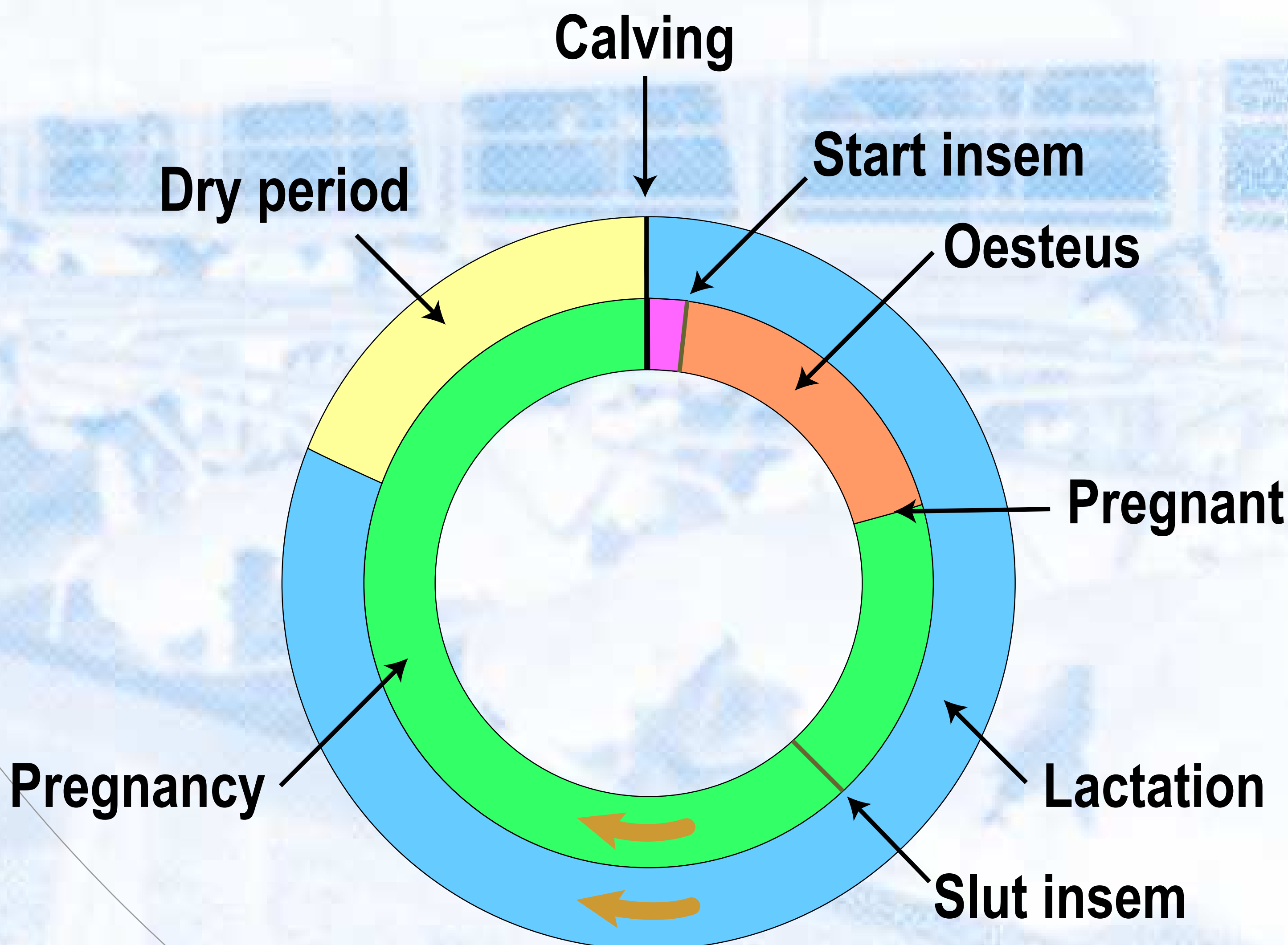
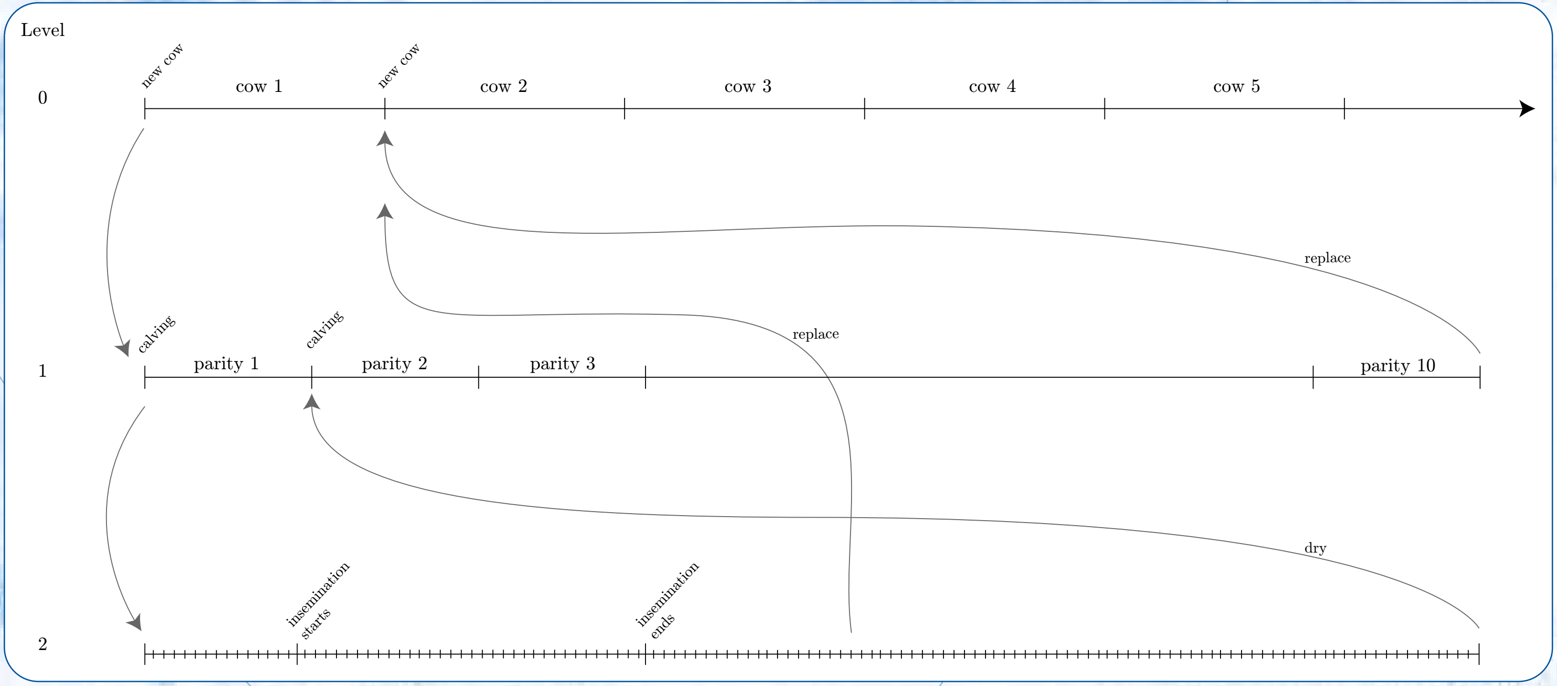


Dairy model overview

- Formulate a hierarchical MDP (HMDP) based on lactation cycles of the cow.
- Infinite time-horizon, Daily stages, 3 levels
- Decisions Replace, Keep and Dry
- Maximize the discounted net present reward of the cow

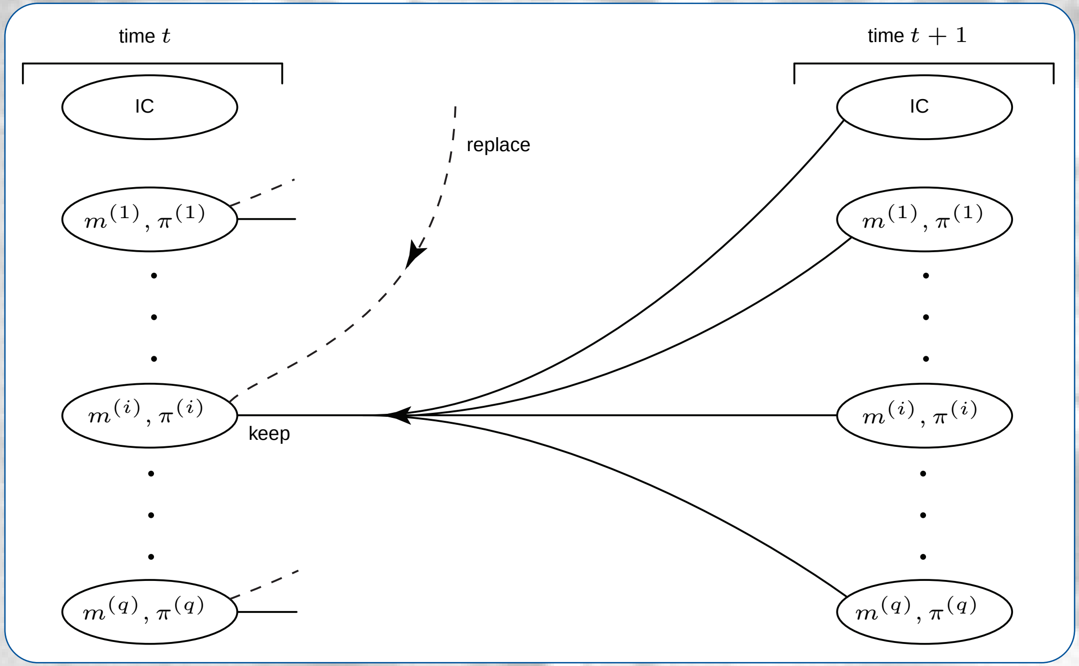


State variables

State variables are

- Dry week π (determine the length of the lactation)
- State variables related to the milk yield, i.e. the mean m of the latent variables (A, X) in the SSM (discretized)

A state in the HMDP consists of a combination of the state variables + IC state.



Rewards and transition probabilities

Transition probabilities are a found using

- ☞ The SSM milk yield model
- ☞ A reproduction model
- ☞ An IC model

- ☞ The calf
- ☞ Beef
- ☞ Feeding and treatment costs

Rewards and transition probabilities

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The net reward is a combination of

- ☞ Milk yield
- ☞ The calf
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SSM models

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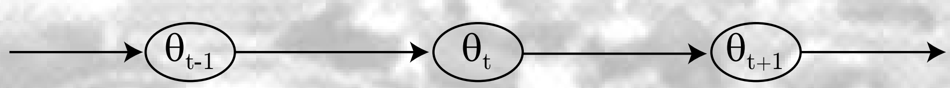

- Latent process evolves as a first order Markov process.

$$\theta_t = G\theta_{t-1} + \omega_t, \;\; (\theta_t \mid \theta_{t-1}) \sim N\left(G\theta_{t-1}, W\right)$$

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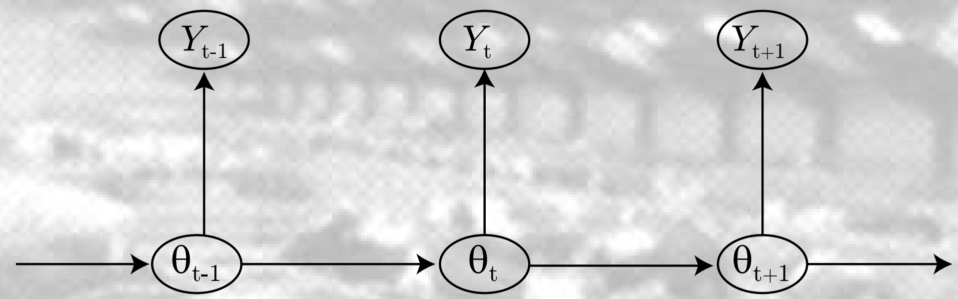

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$$Y_t = F'\theta_t + \nu_t, \ \ (Y_t \mid \theta_t) \sim N\left(F'\theta_t, V\right)$$

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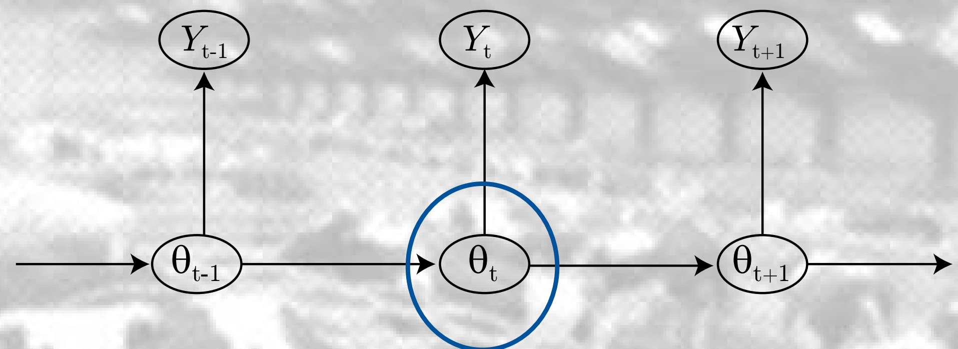

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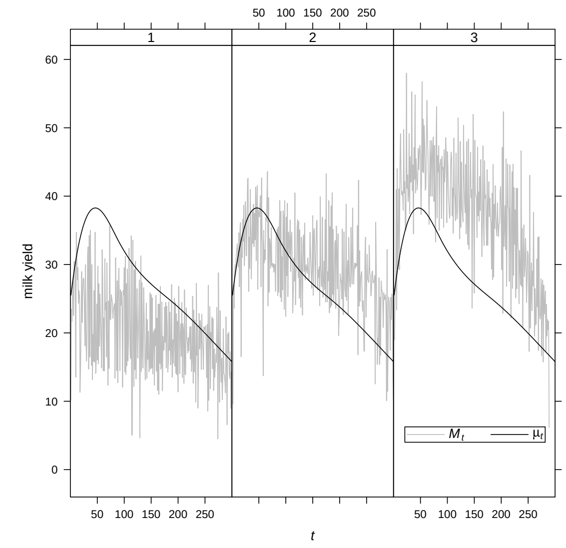
Milk yield SSM

Observed milk yield intensity

$$M_{tc} = \mu_t + A_c + X_{tc} + \nu_{tc}$$



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Subtract herd effect (remove index c)

$$Y_t = M_t - \mu_t = F'\theta_t + \nu_t$$

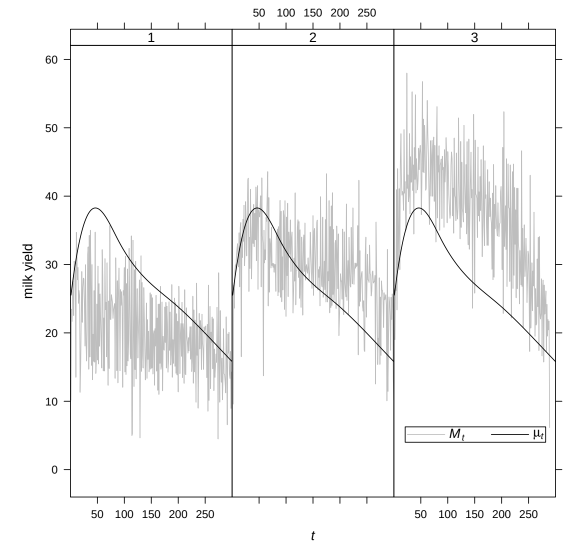
$$= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} A \\ X_t \end{pmatrix} + \nu_t$$

$$\theta_t = G\theta_{t-1} + \omega_t$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} A \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ \epsilon_t \end{pmatrix}$$

where

$$(\theta_t \mid Y_0, \dots, Y_t) \sim N(m_t, C_t)$$



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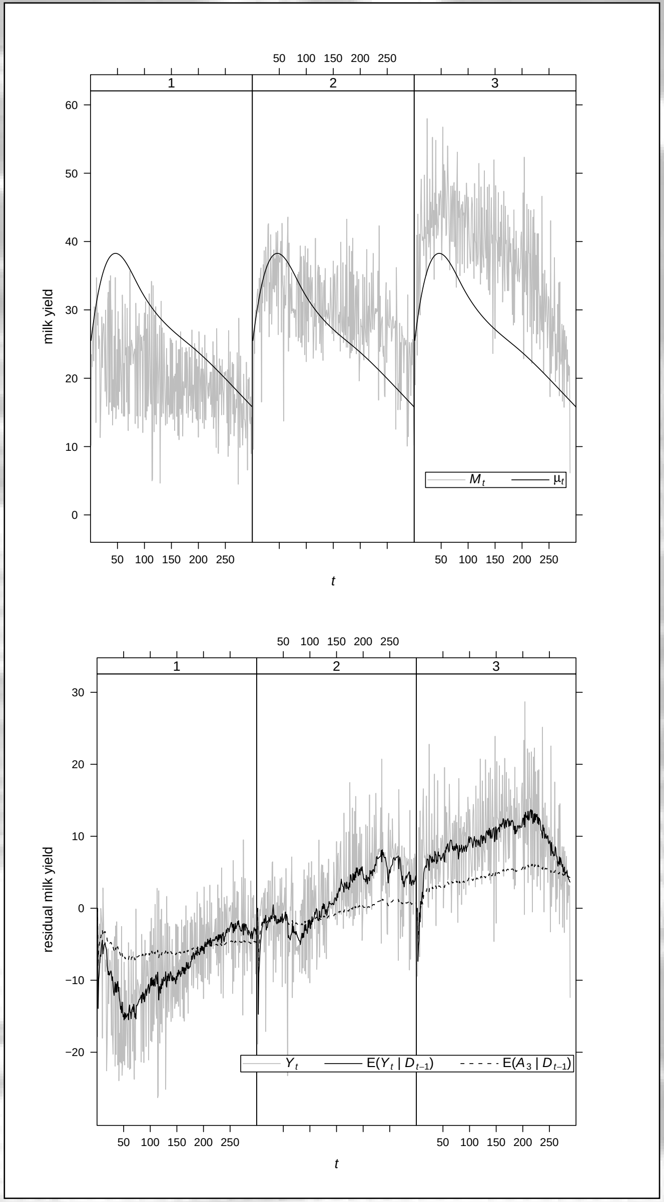
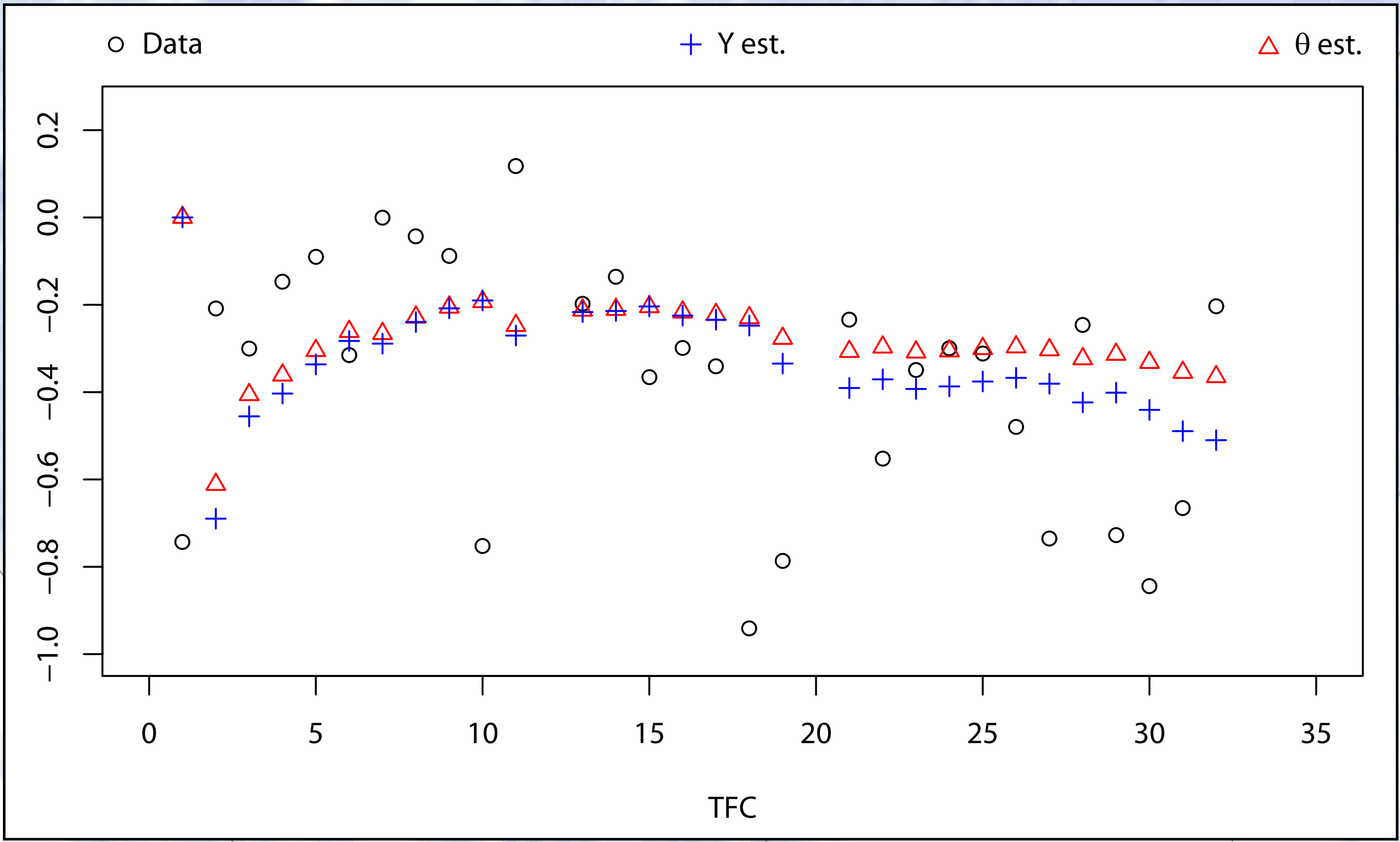


Applying the Kalman filter

D_{t-1}: data up to time $t-1$. Fact:

$$(\theta_{t-1} \mid D_{t-1}) \sim N(m_{t-1}, C_{t-1})$$



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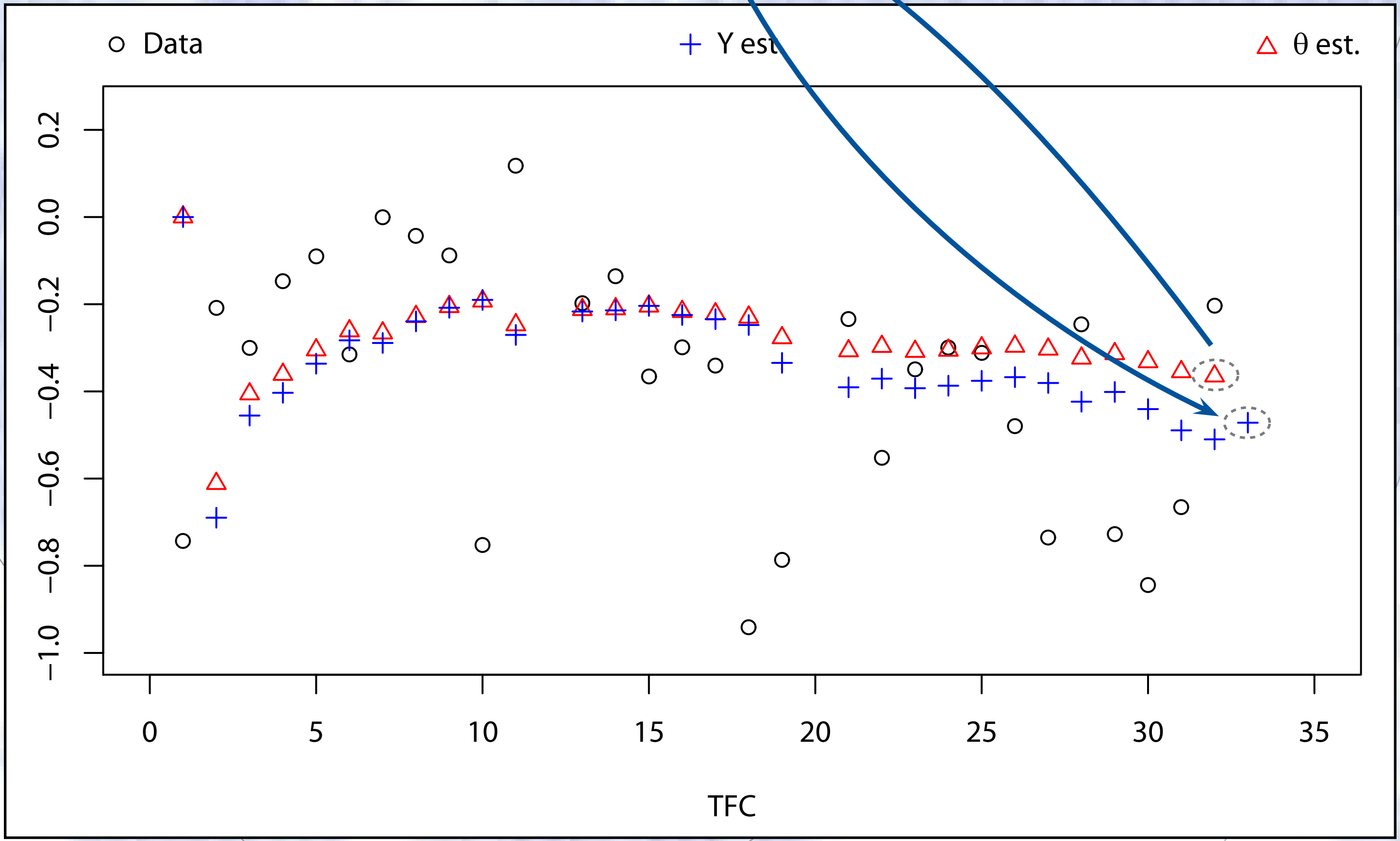
Given $(\theta_{t-1} \mid D_{t-1}) \sim N(m_{t-1}, C_{t-1})$ we have that
$(Y_t \mid D_{t-1}) \sim N(f(m_{t-1}), Q_t)$



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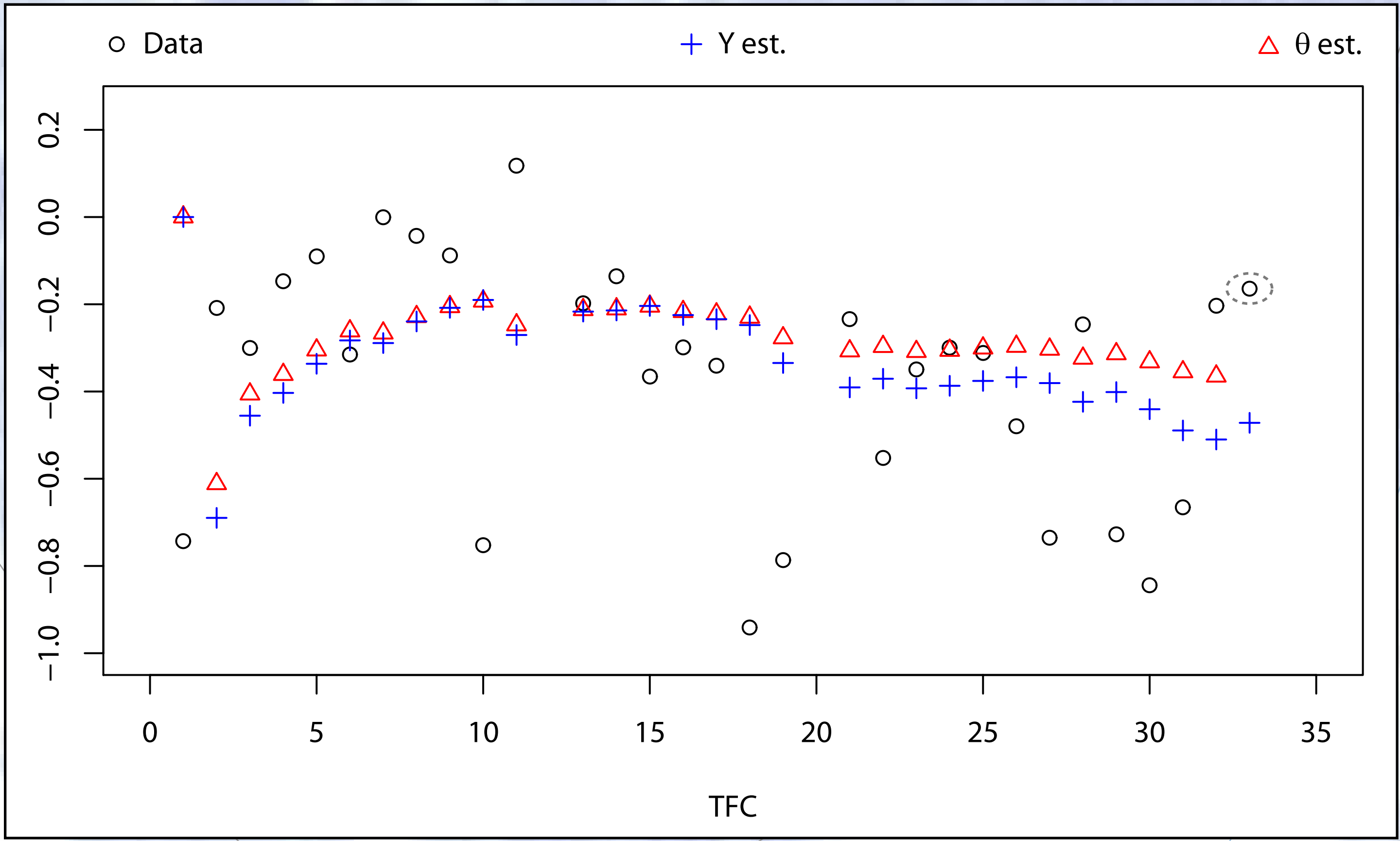

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Given m_{t-1} and Y_t we have that

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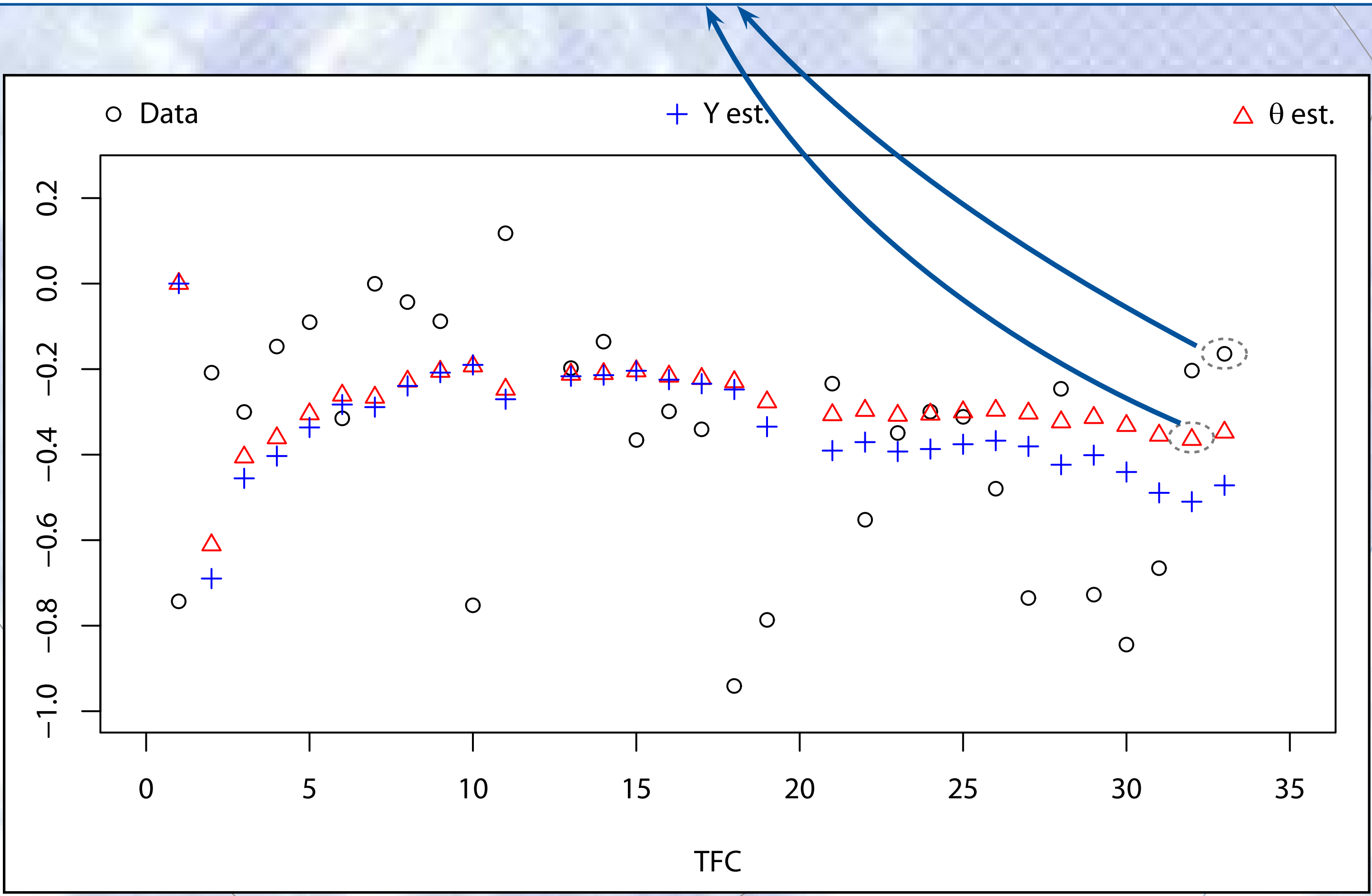

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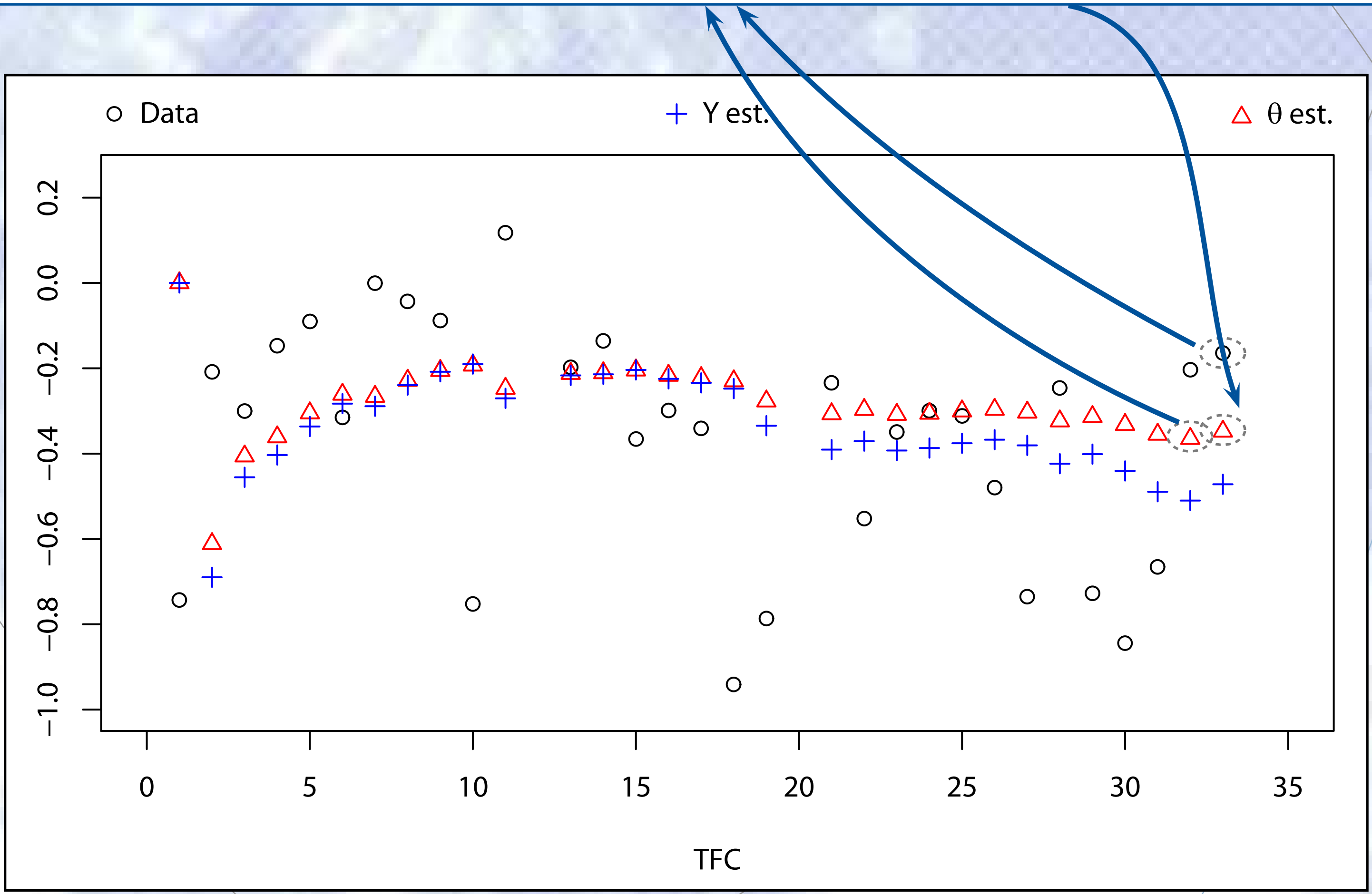

Applying the Kalman filter

Repeat at next time-instance

- Can find $P(m_{t+1} \mid m_t)$ if store the mean m_t and variance C_t in each state.
- Discrete states $\rightarrow$ discretize m_t with $\{\tilde{m}^{(1)}, \ldots, \tilde{m}^{(q)}\}$ and calculate $P(\tilde{m}_{t+1}^{(i)} \mid \tilde{m}_t^{(j)})$
- Discretization can be done uniform or non-uniform. ($m_t = (E(A_t), E(X_t))$).

Embedding the SSM into a MDP

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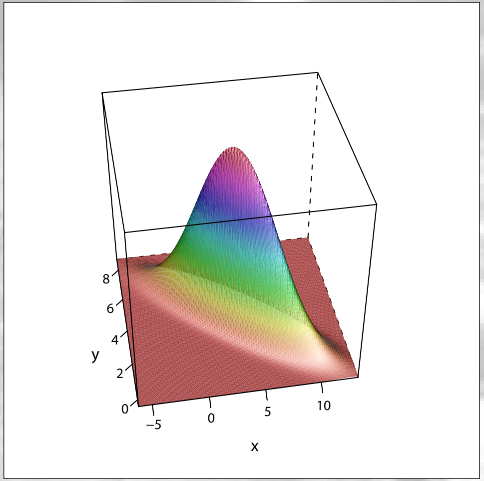
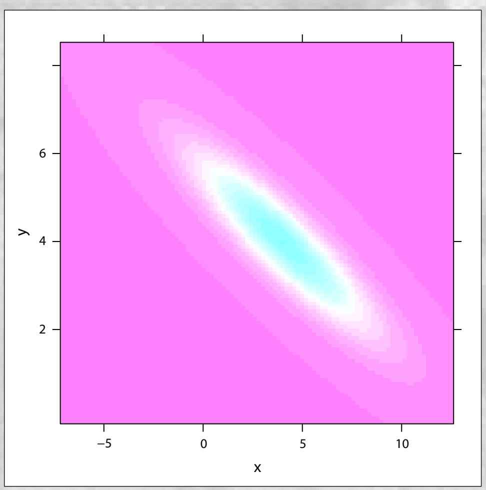
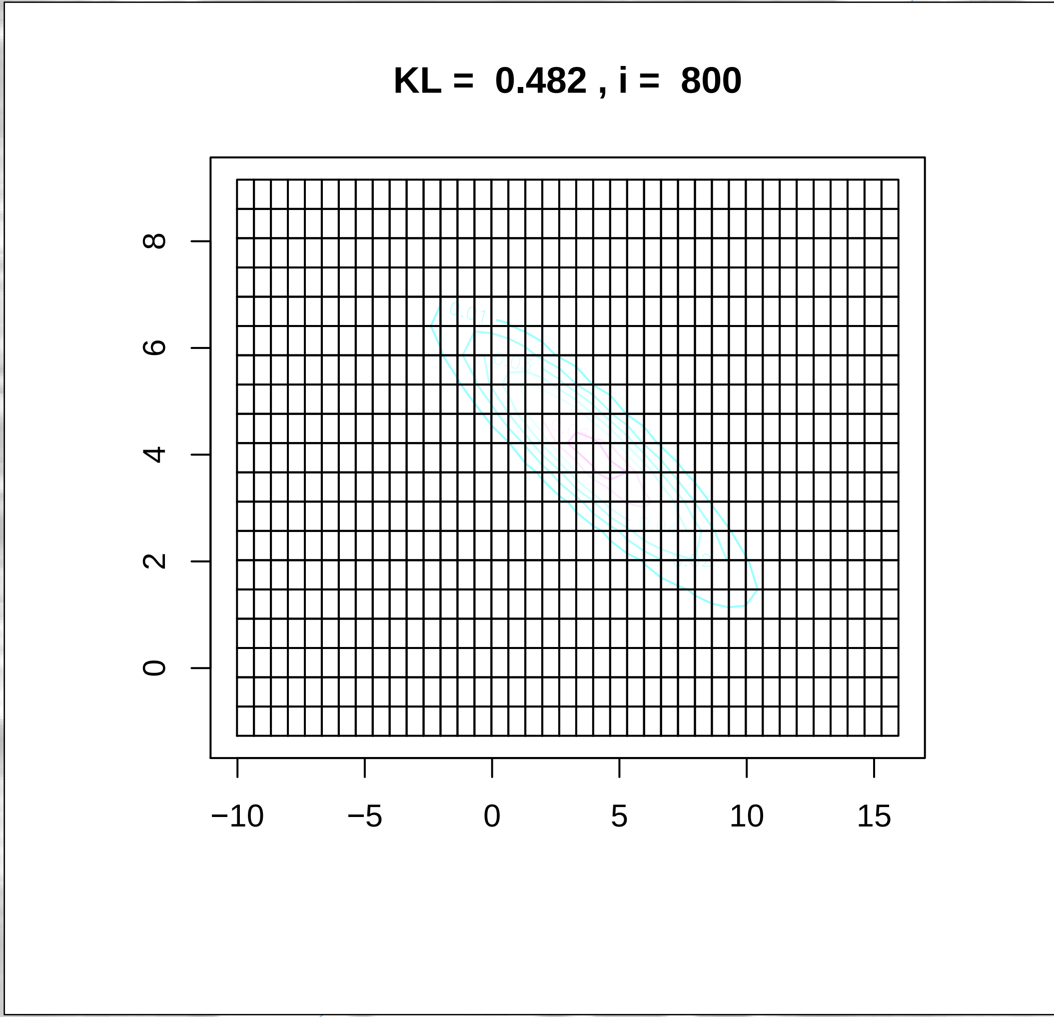
Discretize every variable separately (many states, independent of $\tilde{m}^{(\cdot)}$).

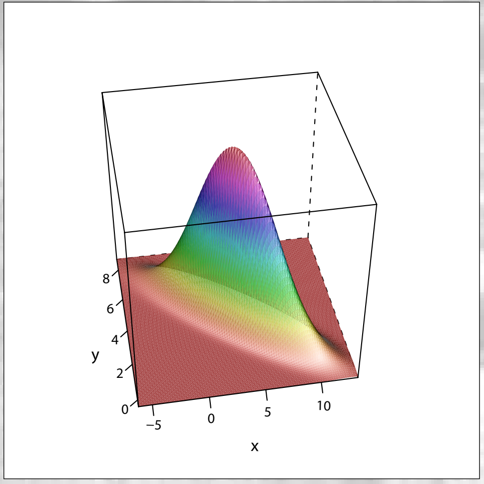


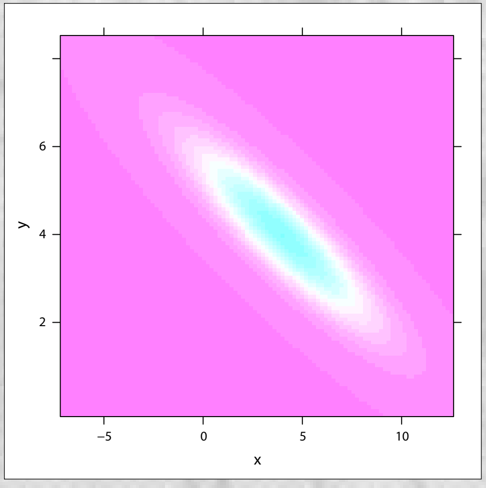


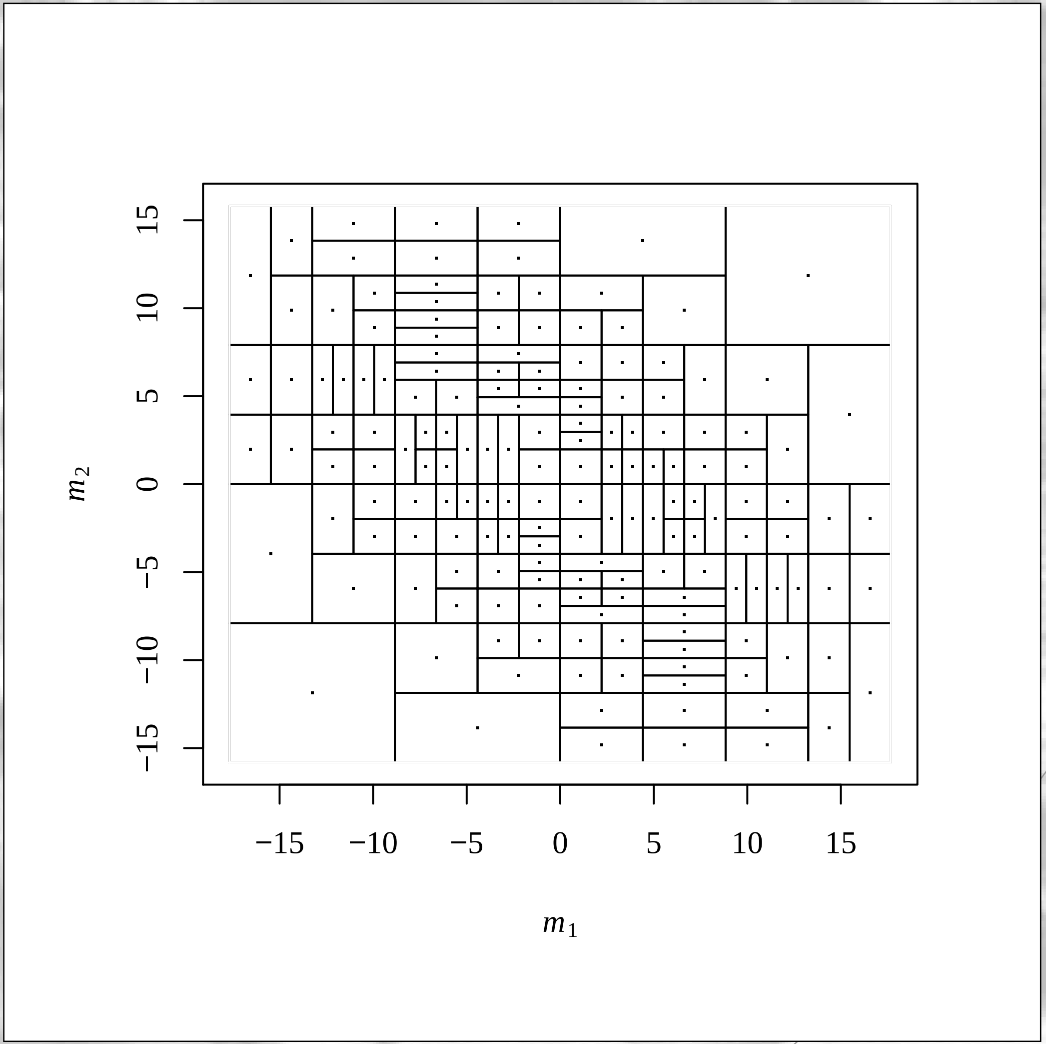


Non-uniform discretization

Discretize the regions of the density (fewer states, dependent on $\tilde{m}^{(\cdot)}$).







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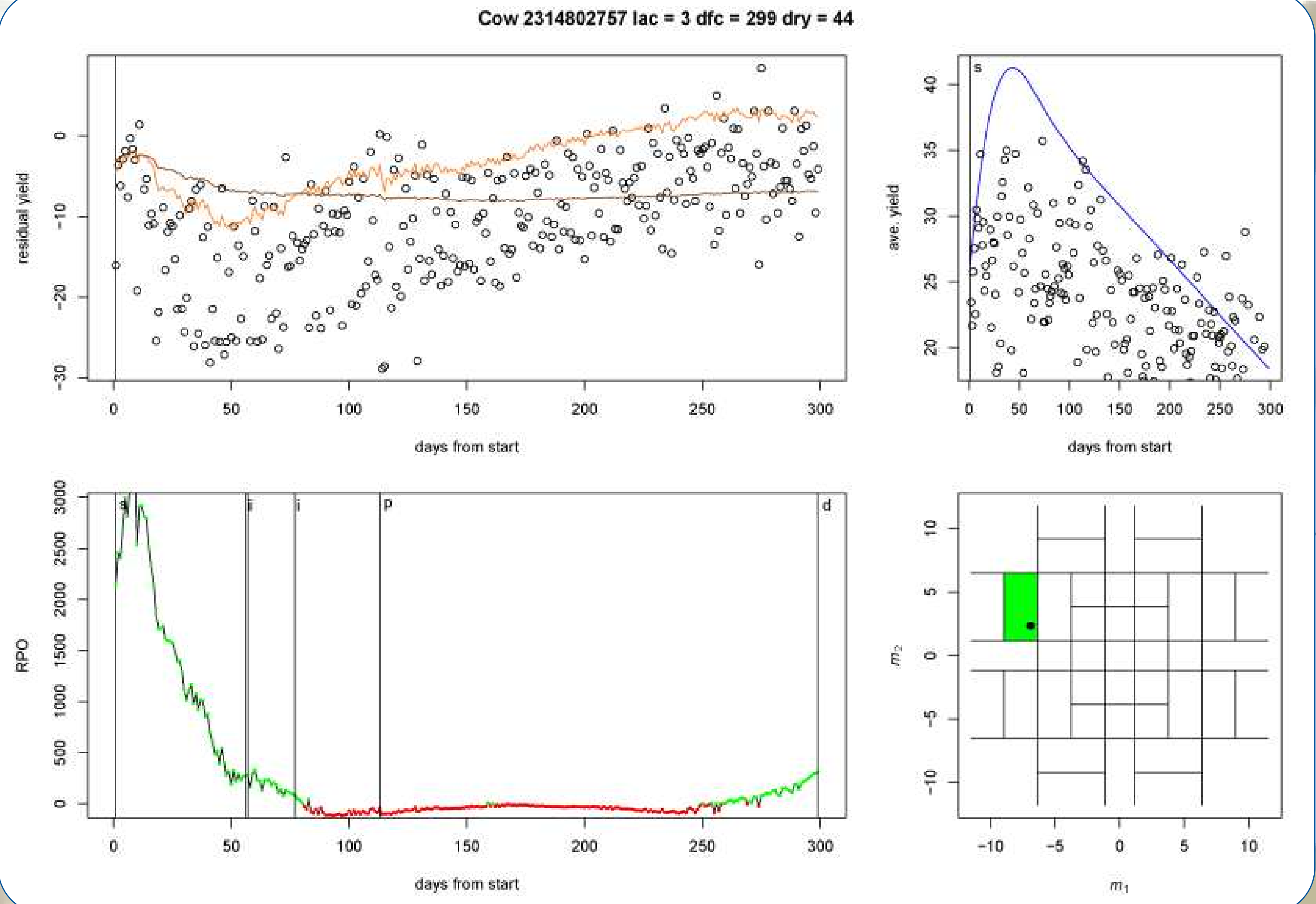
Cow 2314802757 lac = 3 dfc = 299 dry = 44
residual yield
days from start
ave. yield
s
days from start
RPO
s
i
i
p
d
days from start
m_2
m_1

Status

- Model running (Linux) using MLHMP Java library.
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Extensions

Background

Overview

MDP intro

Dairy HMDP Model

State space model

Embedding the SSM

HMDP results

Status/future work

→ Status

→ Extensions