Optimal replacement policies for dairy cows based on daily yield measurements

Case example: Markov decision processes

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Biosens II

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- Biosens II: Improved monitoring and management of dairy production based on on-farm biosensors
- Goal: Better detection of oestrus and illnesses
- Focus on biomarkers in milk (progesterone, LDH, yield, etc.)
- Commercial partner Lattec I/S (FOSS A/S and DeLaval AB)
- Five year project (2007-2011). Budget ≈5 mill EUR
- Commercial product Herd Navigator™ based on Biosens project (`www.herdnavigator.com`)

Project 2.3: Economic value of the dairy cow

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Find optimal strategy for each cow w.r.t. replacement, treatment and reproduction (economic point of view).

- Many papers about the dairy cow replacement problem but limited use in pratice. Reasons could be:
  - Often large and complex models.
  - Parameters in the model must be estimated, i.e. data collection frameworks at herd level must exist.
  - Stage length: one month up to a year → no assistance when to inseminate, treat or cull the cow in the current month.
- Bio-sensors and cow specific traits/interventions exists in modern dairy herds → parameters can be estimated on a daily basis.

Develop MDP with daily stages based on daily yield measurements.

Problem, models and results

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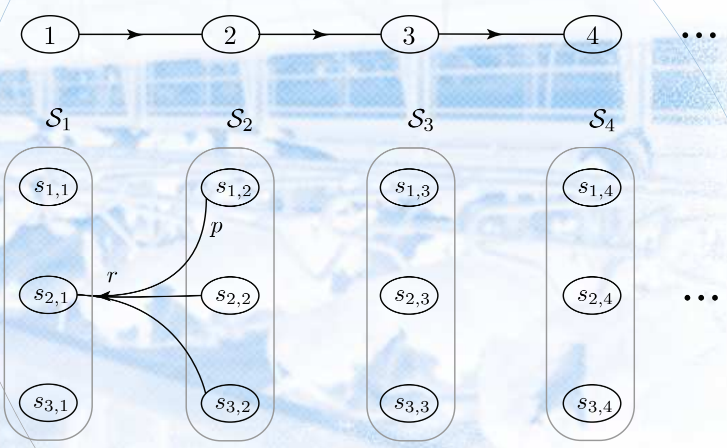
Problem
- Assign an economic value to a dairy cow during lactation
- Calculate the optimal replacement strategy based on the economic value
- Assume daily yield measurements available

Models
- Use a state space model (SSM) for predicting daily milk yield
- Use a Markov decision process (MDP) for calculating the optimal strategy with the SSM embedded

Results
- A strategy saying whether to replace or keep the cow given its current state
- An economic value of the cow and forecast of the yield.

What is an MDP?



What is an MDP?

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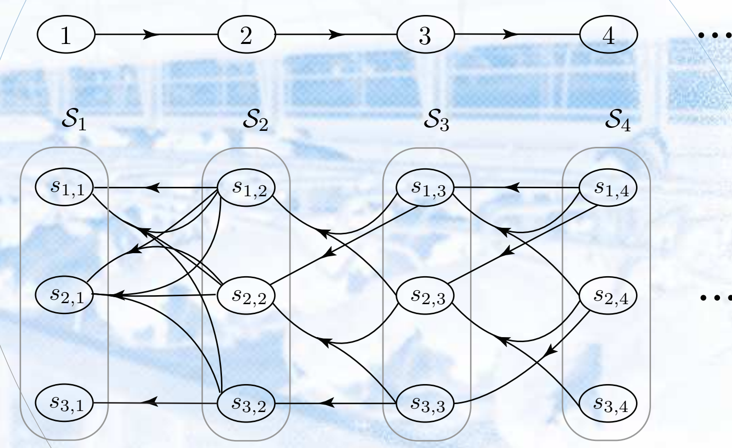

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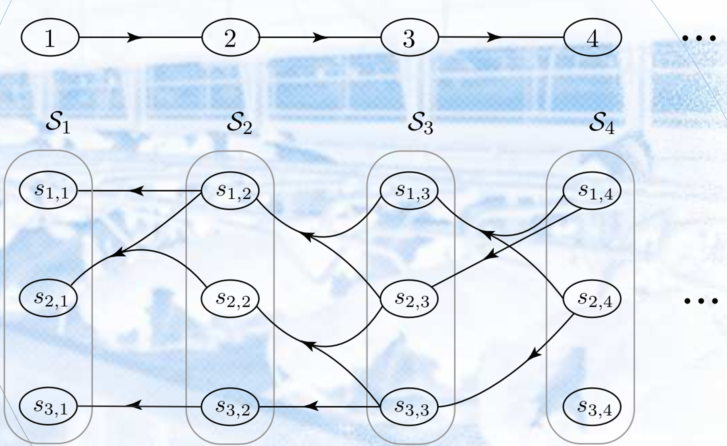

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Hierarchical MDP (HMDP)

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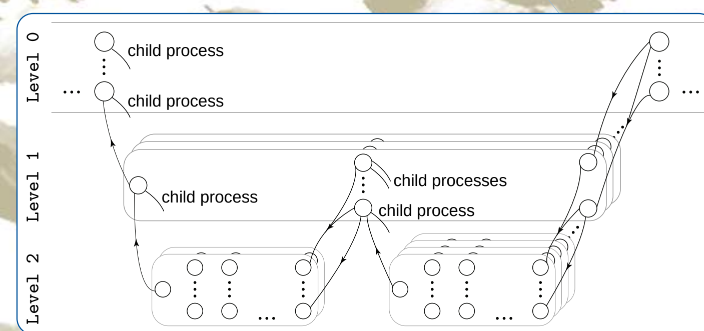

Lactation cycle of the cow

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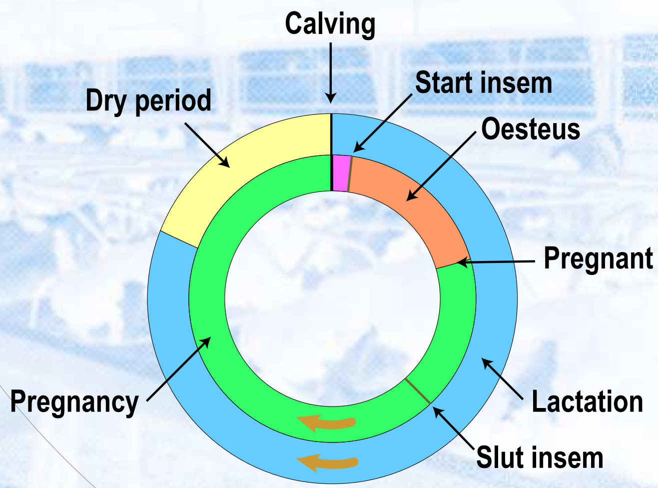

Dairy model overview

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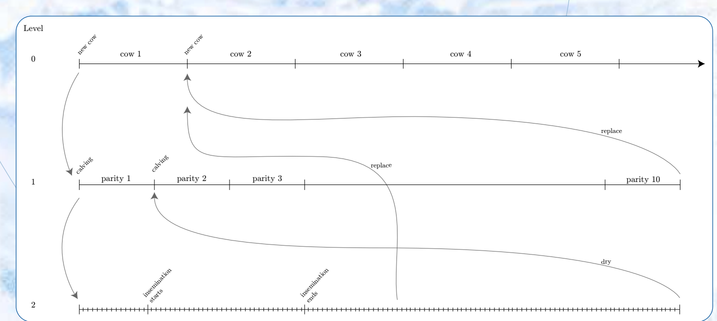
- Formulate a hierarchical MDP (HMDP) based on lactation cycles of the cow.
- Infinite time-horizon, Daily stages, 3 levels
- Decisions Replace, Keep and Dry
- Maximize the discounted net present reward of the cow



Rewards and transition probabilities

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Transition probabilities are a found using

- The SSM milk yield model
- A reproduction model
- An IC model

The net reward is a combination of

- Milk yield
- The calf
- Beef
- Feeding and treatment costs

SSM models

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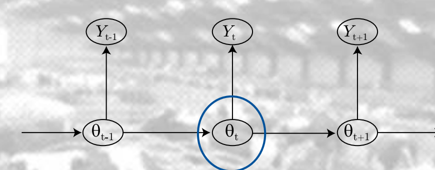
- ... or dynamic linear models are models of phenomena evolving in time e.g. blood pressure and milk yield.



- Latent process evolves as a first order Markov process.

$$\theta_t = G\theta_{t-1} + \omega_t, \quad (\theta_t \mid \theta_{t-1}) \sim N\left(G\theta_{t-1}, W\right)$$

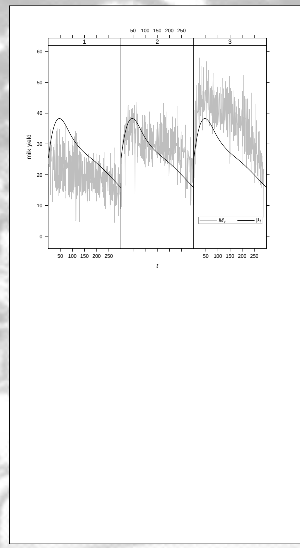
- Y_t are observations which we model as a function depending on θ_t.

$$Y_t = F'\theta_t + \nu_t, \quad (Y_t \mid \theta_t) \sim N\left(F'\theta_t, V\right)$$

Milk yield SSM

Observed milk yield intensity

$$M_{tc} = \mu_t + A_c + X_{tc} + \nu_{tc}$$



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Milk yield SSM

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Observed milk yield intensity

$$M_{tc} = \mu_t + A_c + X_{tc} + \nu_{tc}$$

Subtract herd effect (remove index c)

$$Y_t = M_t - \mu_t = F'\theta_t + \nu_t$$

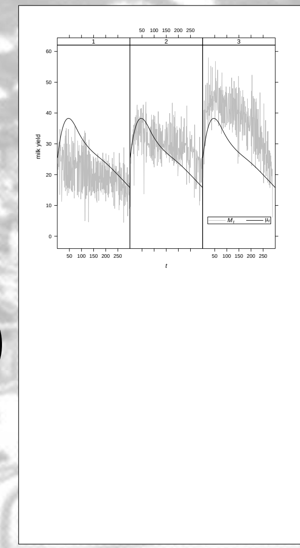
$$= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} A \\ X_t \end{pmatrix} + \nu_t$$

$$\theta_t = G\theta_{t-1} + \omega_t$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} A \\ X_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ \epsilon_t \end{pmatrix}$$

where

$$(\theta_t \mid Y_0, \ldots, Y_t) \sim N(m_t, C_t)$$



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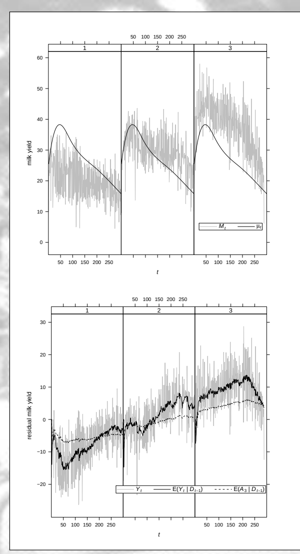

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Applying the Kalman filter

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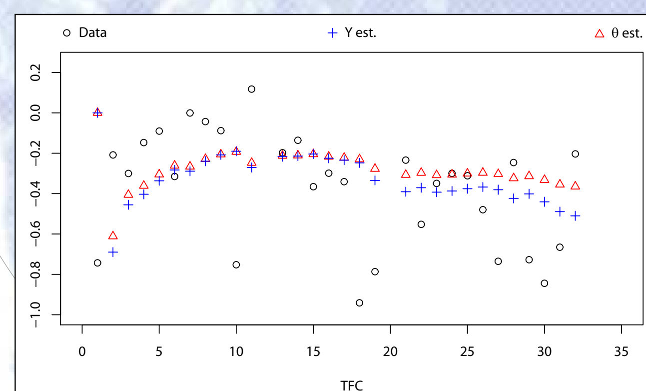
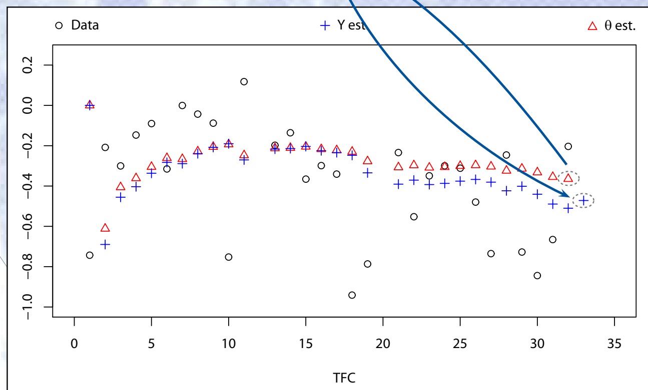
D_{t-1}: data up to time $t-1$. Fact:
$(\theta_{t-1} \mid D_{t-1}) \sim N(m_{t-1}, C_{t-1})$



Applying the Kalman filter

Given $(\theta_{t-1} \mid D_{t-1}) \sim N(m_{t-1}, C_{t-1})$ we have that
$(Y_t \mid D_{t-1}) \sim N(f(m_{t-1}), Q_t)$



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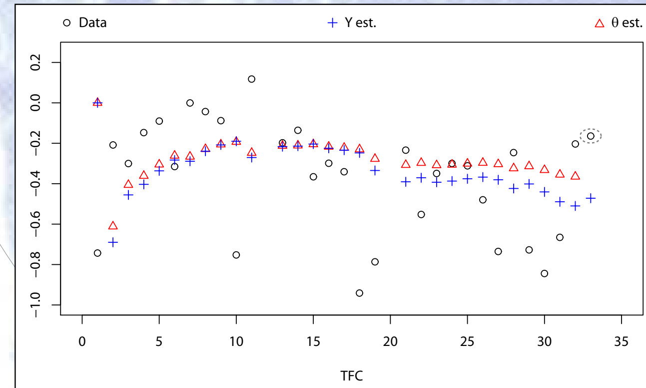

Embedding the SSM into a MDP

- Can find $P(m_{t+1} \mid m_t)$ if store the mean m_t ~~and variance C_t~~ in each state.
- Discrete states → discretize m_t with $\{\tilde{m}^{(1)}, \dots, \tilde{m}^{(q)}\}$ and calculate $P(\tilde{m}_{t+1}^{(i)} \mid \tilde{m}_t^{(j)})$
- Discretization can be done uniform or non-uniform. $(m_t = E(A_t E(X_t)))$.

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- Can find $P(m_{t+1} \mid m_t)$ if store the mean m_t ~~and variance C_t~~ in each state.
- Discrete states $\longrightarrow$ discretize m_t with $\{\tilde{m}^{(1)}, \ldots, \tilde{m}^{(q)}\}$ and calculate $P(\tilde{m}^{(i)}_{t+1} \mid \tilde{m}^{(j)}_t)$
- Discretization can be done uniform or non-uniform. $(m_t = (E(A_t), E(X_t)))$.

Uniform discretization

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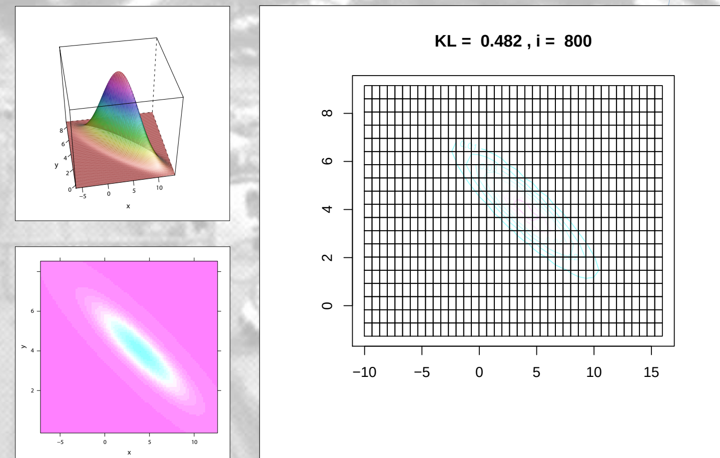
Discretize every variable separately (many states, independent of $\tilde{m}^{(\cdot)}$).



Non-uniform discretization

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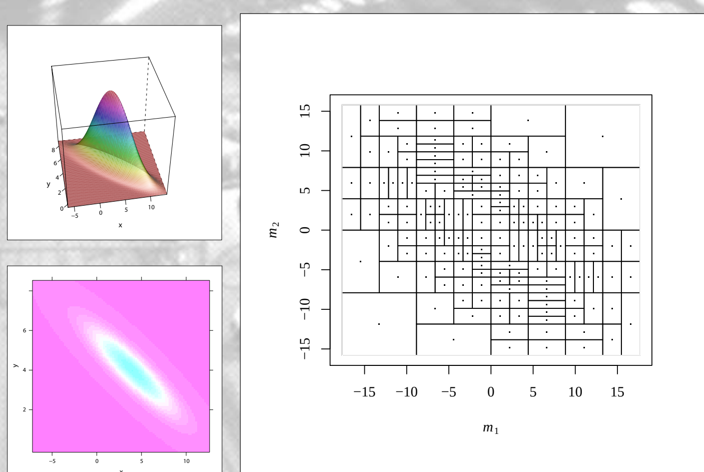
Discretize the regions of the density (fewer states, dependent on $\tilde{m}^{(\cdot)}$).



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- Model running (Linux) using MLHMP Java library.
- Manuscript accepted in JDS.
- Currently working on evaluating different reproduction strategies in the model. Challenge: Number of state variables (complexity)
- Other spinoffs: A MDP and dairy package in R.