Markov decision processes to model livestock systems*

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Abstract: Livestock farming problems are often sequential in nature. For instance at a specific time instance the decision on whether to replace an animal or not is based on known information and expectation about the future. At the next decision epoch updated information is available and the decision choice is re-evaluated. As a result Markov decision processes (MDPs) have been used to model livestock decision problems over the last decades. The objective of this chapter is to review the increasing amount of papers using MDPs to model livestock farming systems and provide an overview over the recent advances within this branch of research. Moreover, theory and algorithms for solving both ordinary and hierarchical MDPs are given and possible software for solving MDPs are considered.

1 Introduction

Mathematical models for livestock farming systems have been used since the fifties. Examples of techniques used include deterministic optimization such as linear programming (for an early example, see Fisher and Schruben, 1953) and dynamic programming (with White, 1959, as one of the first applications to livestock farming) as well as stochastic models based on Monte Carlo simulation (e.g. Sørensen et al, 1992) and Markov decision processes (*MDPs*).

The nature of livestock systems differ from other industrial systems. Compared to, e.g., modeling the state of a machine, modeling the state of, e.g., a cow is more complex. First, the traits of an animal is harder to estimate and animals like humans differ, i.e., the variance between animals is much higher and it is harder to determine which state the animal is in. Second, livestock systems have a cyclic nature. In most cases an animal is inserted into the herd and after some cyclic periods (lactations, parity, feeding cycle) replaced with a new animal. Decisions regarding which cycle and when to replace the animal within the cycle have to be taken. Finally, often

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the supply of animals is not unlimited, e.g., a cow cannot be replaced if we do not have a heifer available. These three characteristics have also been referred to as the uniformity, reproductive cycle, and availability features of livestock systems (Ben-Ari et al, 1983).

Livestock farming is often sequential in nature. For instance at a specific time instance the decision on whether or not to replace an animal is based on observed information and expectation about the future. At the next decision epoch updated information is available and the decision choice is re-evaluated. Since random variation is a core property of a livestock system, MDPs have often been used to model livestock decision problems over the last decades (see Kristensen, 1994, for an overview). At a specified point in time, the decision maker observes the state of a system and makes a decision. The decision and the state of the process produce two results: the decision maker receives an immediate reward (or incurs an immediate cost), and the system evolves probabilistically to a new state at a subsequent discrete point in time. At this subsequent point in time, the decision maker faces a similar problem. However, the observed state may be different from the previously observed state. The goal is to find a policy of decisions (dependent on the observation of the state) that maximizes, for example, the expected discounted reward.

In the MDP the state of the animal is defined by a set of state variables, each representing a trait relevant for the livestock system under consideration, e.g. for a dairy cow state variables could be milk yield level, lactation number, days in milk, reproductive status etc. It is assumed that the value of the state variable belongs to a finite set of levels/classes that represent the value of the trait. Often a trait is continuous and must be discretized into a set of levels. If we consider a realistic number of levels we may face the problem known as the "curse of dimensionality": the number of possible states grows exponentially with the number of state variables (the state space is often formed as the cartesian product of the number of levels of each of the state variables). This is one of the major drawbacks of using a MDP to model a livestock system.

Hierarchical MDPs (*HMDPs*) are an attempt to decompose the state space and to reduce the number of states in the MDP. The model is a series of finite time MDPs built together into one MDP called the founder process. As a result, the age of the animal can be omitted in the state space compared with an ordinary MDP model. Moreover, it takes into account that the production is cyclic. When a replacement occurs, not just a regular state transition takes place but rather the process (life cycle of the replacement animal) is restarted. HMDPs were first considered by Kristensen (1988) assuming 2 levels in the HMDP. Later, Kristensen and Jørgensen (2000) extended the methodology to multi-level HMDPs such that MDPs can be built together at multiple levels. Note that an HMDP is an infinite-stage MDP with parameters defined in a special way, but nevertheless in accordance with all usual rules and conditions relating to such processes. The basic idea of the hierarchic structure is that stages of the process can be expanded to the so-called child processes, which again may expand stages further to new child processes leading to multiple levels. Even though that HMDPs may help to reduce the number of state variables, the curse of dimensionality is still a problem.

In most papers an MDP is used to model a single animal and its successors (single-component). Hence herd constraints (heifers, feed, milk-quota, etc.) are not taken into account. To represent the whole herd a multi-component MDP has to be considered as discussed in Ben-Ari and Gal (1986) and Kristensen (1992). The multi-component model is based on single-component MDPs representing a single animal and its future successors. However, the model is far too large for optimization in practice. Therefore, the need for an approximate method emerged, and a method called *parameter iteration* was introduced by Ben-Ari and Gal (1986) and later modified by Kristensen (1992) to whom reference is made for details. To the authors' knowledge the parameter iteration method has only been applied under a constraint of a limited supply of heifers Kristensen (1992).

The state of an MDP must be directly observable. Since the state in the model represents the present traits of the animal in question, this means that the traits are assumed to be well defined and directly observable. This is not always the case. Traits of an animal vary no matter whether we are considering the milk yield of a dairy cow or the litter size of a sow. Moreover, it is not obvious to what extent the observed trait is a result of a permanent property of the animal or a temporary random fluctuation. Most often the observed value is the result of several permanent and random effects. This problem can be solved by modeling the trait as a stochastic process and embedding the parameters of the process into the MDP instead of the observed value of the trait. The technique is referred to as *Bayesian updating*. As observations are done, the Bayesian approach is used to increase the knowledge on the true value of the trait. The technique was first used in practise by Kennedy and Stott (1993) for milk yield and has been described in detail by Kristensen (1993) and generalized in Nielsen et al (2011).

For an MDP to be valid the *Markov property* must be fulfilled. It implies that the state space at any decision epoch (or *stage*) must contain sufficient information for determination of the probability distribution of the state to be observed at next decision epoch. In a straight forward formulation of a decision problem this is rarely the case, and various tricks must be used in order to make the process Markovian. The most common trick is to include *memory variables* in the state space (for instance the milk yield of previous lactation(s) in dairy cow models). This approach has been used in numerous models in practice. A more elaborate approach is to use Bayesian updating to estimate latent traits (for instance an abstract milk yield capacity of a dairy cow) as observations are done over time.

The objective of this chapter is to review the increasing amount of papers using MDPs to model livestock farming systems and provide an overview over the recent advances within this branch of research. Moreover, theory and algorithms for solving both ordinary and hierarchical MDPs are given and possible software for solving MDPs are considered. The chapter provides and updated overview compared to the latest survey (Kristensen, 1994) which is almost 20 years old. The authors have tried to include all peer-review articles using MDPs to model livestock systems which resulted in more than 80 papers in total. Some very old applications (mainly from the sixties and seventies) have been omitted in the overview. Most of those early applications were deterministic and some of them were published in research reports which are not available online . Readers who are interested in those papers are referred to Kennedy (1986) who gives an overview of applications until the early eighties.

The chapter is organized as follows. In Section 2 a short introduction to ordinary MDPs and hierarchical MDPs is given and algorithms for optimizing the process are described. Next, a survey over papers using MDPs applied to cattle farming problems is given in Section 3. Dairy production is the most successful area on which MDPs have been applied. The chapter is continued in Section 4 with a survey over papers within the area of pig production. Finally, a few papers which lies outside these two areas are considered in Section 5. Software for solving both ordinary and hierarchical MDPs are discussed in Section 6. At last conclusions and directions for future research are discussed in Section 7.

2 Methodology

We briefly introduce the methodology of MDPs and describe the different algorithms which can be used to find an optimal policy under different criteria. Many papers using MDPs to solve livestock problems consider a stochastic process where the length of a stage is not constant. This is actually an extension of the MDP methodology (where a constant stage length is assumed), referred to as a semi MDP (Tijms, 2003). However, due to the use of the term MDP instead of a semi MDP in the past we will stick to this. Indeed, throughout the rest of the paper we will use the term MDP for both ordinary and hierarchical (semi) MDPs and explicit write ordinary or hierarchical if needed.

2.1 Finite-horizon Markov decision processes

We consider an ordinary finite-horizon MDP with N *stages*. At stage n the system occupies a *state* belonging to the finite set of system states S_n. Given that the decision maker observes state $s \in S_n$ at stage n, he must choose an *action* a from the set of finite allowable actions $A_{s,n}$ generating an immediate *reward* $r_s^a(n)$. Let $t_s^a(n)$ denote the expected length of stage n, i.e., the time until the system evolves probabilistically to a new state (decision epoch) and $\beta_s^a(n)$ the corresponding *discount rate* of the stage. Note that if α denotes the interest rate per time unit, and the stage length is L, then the discount factor is $\exp(-\alpha L)$ if we assume continuous compounding or $1/(1+\alpha)^L$ if we assume periodic compounding. Let $p_{s\hat{s}}^a(n)$ denote the *transition probabilities* of observing state $\hat{s} \in S_{n+1}$ at stage $n+1$ given state s and action a.

A *policy* δ is a function that assigns to each state s a fixed action $a = \delta(s)$, i.e., a policy provides the decision maker with a plan of which action to take given stage and state. Under a given policy we write $r_s^a(n)$, $t_s^a(n)$ and $p_{s\hat{s}}^a(n)$ as $r_s^\delta(n)$, $t_s^\delta(n)$ and $p_{s\hat{s}}^\delta(n)$, respectively.

Let X_n denote the state of the system at the n'th decision epoch. Under a finite time-horizon the *total expected discounted reward* criterion may be relevant when consider livestock problems:

$$h(\delta) = \mathbb{E}\left(\sum_{n=1}^{N} r_{X_n}^{\delta}(n) \prod_{i=1}^{n-1} \beta_{X_i}^{\delta}(i)\right), \tag{1}$$

where the product is the total discount factor need to discount the reward at stage n back to stage 1. Moreover, if no discounting is used ($\alpha = 0$) then (1) calculates the *total expected reward.* It is assumed that no decision is taken at decision epoch N, i.e. a deterministic dummy action $a_N = \delta(X_N)$ is taken. The reward $r_{X_N}^{a_N}(N)$ is often referred to as the *terminal or salvage reward.*

Having introduced the notation for an MDP, we are also able to give a formal definition of the Markov property mentioned in the introduction. The Markov property is satisfied in an MDP if and only if

$$\mathrm{P}_a(X_{n+1}|X_n) = \mathrm{P}_a(X_{n+1}|X_n,\ldots,X_1) = p_{X_nX_{n+1}}^a, \forall n < N, X_n \in S_n, a \in A_{X_n,n},$$

where P_a denotes the probability function under the decision a. In words it means that the state at next stage is only allowed to depend on the present state and action. Any other historical information is of no relevance. It is essential for the correctness of the results from an MDP that this property is satisfied.

An optimal policy maximizing (1) can be found using the following *Bellman equations*, Bellman (1957):

$$v_n(s) = \begin{cases} \max_{a \in A_{s,n}} \left\{ r_s^a(n) + \beta_s^a(n) \sum_{\hat{s} \in S_{n+1}} p_{s\hat{s}}^a(n) v_{n+1}(\hat{s}) \right\} & n < N \\ r_s^{a_N}(N) & n = N \end{cases}, \tag{2}$$

where $v_n(s)$ is the total expected discounted reward in state s at stage n under the optimal policy until the process terminates. Equations (2) shows that the optimal policy can be found by analyzing a sequence of simpler inductively defined single-stage problems. This is often referred to as *value iteration*.

2.2 Infinite-horizon Markov decision processes

A situation where the stage of termination is unknown (or at least far ahead) is usually modeled using an infinite planning horizon ($N = \infty$). Given that the process is time homogeneous, i.e., the states and actions are independent of stage number and the policy stationary (constant over stages), we can drop the index n from the notation given in Section 2.1. Criterion (1) can still be considered (now an infinite sum) and will converge toward a fixed value when increasing N if discount rates are less than one.

Let $Z(t)$ denote the total reward incurred until time t and assume that the MDP is unichain (see Tijms (2003) for a formal definition). As an alternative criterion we may consider the *average reward per time unit*:

$$g(\delta) = \lim_{t \to \infty} \frac{Z(t)}{t} = \frac{\sum_{s \in S} \pi_s^\delta r_s^\delta}{\sum_{s \in S} \pi_s^\delta t_s^\delta} \tag{3}$$

where π_s^δ are the limiting state probabilities or *equilibrium distribution probabilities* given policy δ. Other criteria such as the *average reward per physical output* can also be considered and are defined as in (3) by redefining t_s^a as the physical output instead. For instance, Nielsen et al (2004) maximize the average reward per steer. Furthermore, if all stages have equal length the denominator of (3) equals one and (3) reduces to the well-known formula for an ordinary MDP.

Various optimization techniques can be used to find the optimal policy such as value iteration, policy iteration and linear programming. We will restrict ourselves to the first two here since linear programming has only been used in two of the papers reviewed.

Value iteration can be used to approximate the optimal policy. It has been used in the majority of papers since it is relatively straightforward to implement the algorithm. Moreover, the algorithm is good for solving large-scale MDP problems since there is no need for solving a large set of equations simultaneously. However, the number of iterations is problem dependent and typically increases in the number of states of the problem under consideration. The value iteration algorithm is given in Figure 1. The algorithm is initialized in Step 0 where a pre-specified small accuracy number ε is chosen. Next, we use the recursive equations to update $v_s(n)$, which under

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Step 0: Set $v_s(0)$ such that $0 \le v_s(0) \le \min_{a\in A_s}\{r_s^a/t_s^a\}, \forall s \in S$. Choose a number $\varepsilon > 0$, set n←0 and
  $\tau = \min_{s\in S, a\in A_s}\{t_s^a\}$ (under criterion (3)).
Step 1: For each $s \in S$ compute $v_s(n)$ using the recursive equation in Table 1 and let $\delta$ be the
  policy whose actions maximize $v_s(n)$.
Step 2: Compute the bounds $m_n = \min_{s\in S}\{v_s(n) - v_s(n-1)\}$ and $M_n = \max_{s\in S}\{v_s(n) - v_s(n-1)\}$
Step 3: If the condition in Table 1 is statisfied then stop; otherwise set n← n+1 and go to
  Step 1.
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Figure 1: Value iteration algorithm for an infinite-horizon ordinary MDP.

Table 1: Equations and expressions to be used in the value iteration algorithm.

Criterion	Step 1 - Recursive equation	Step 3 - Condition
(1)	$v_s(n) = \max_{a\in A_s}\left\{ r_s^a + \sum_{\hat{s}\in S} \beta_s^a p_{s\hat{s}}^a v_{\hat{s}}^{\delta}(n-1) \right\}$	$M_n \le \varepsilon$
(3)	$v_s(n) = \max_{a\in A_s}\left\{ \frac{r_s^a}{t_s^a} + (1 - \frac{\tau}{t_s^a}) v_s(n-1) + \frac{\tau}{t_s^a} \sum_{\hat{s}\in S} p_{s\hat{s}}^a v_{\hat{s}}^{\delta}(n-1) \right\}$	$0 \le M_n - m_n \le \varepsilon m_n$

criterion (1) denotes the total expected discounted reward in state s with n periods left and a terminal cost of $v_s(0)$. Under criterion (3) the recursive equation is based on a data transformation method (see (Tijms, 2003)). This is repeated until the stopping condition is met (Step 3).

Note that if ε is sufficiently small and the same policy is found during several iterations, we may be rather sure that the optimal policy has been found. However, there is no guarantee but for practical purposes the deviation will have no significance. Under criterion (3) the stopping criterion ensures that $0 \le (g^* - g(\delta))/g^* \le \varepsilon$, where g^* denotes the optimal value to (3), i.e., the average reward per time unit $g(\delta) \in [m_n, M_n]$ is at most $100\varepsilon\%$ away from the optimal average reward per time unit. Finally observe that if the time between each decision epoch is constant ($t_s^a = 1$ and $\beta_s^a = \beta$), then the recursive formulas in Table 1 reduces to the well-known formulas for an ordinary MDP. During the years more advanced variants of value iteration algorithms have been developed which provide faster convergence and better stopping conditions. The interested reader is referred to Tijms (2003) and Puterman (1994) for details.

Policy iteration unlike value iteration finds an optimal policy in a finite number of steps. The algorithm is robust in the sense that in general it converges very fast, the number of iterations are independent of the number of states and varies typically between 3 and 15 (Tijms, 2003). However, to use the algorithm $|S|$ linear equations must be solved simultaneously which may be computational costly for large state spaces. The policy iteration algorithm is given in Figure 2. In Step 0 an arbitrary policy is chosen and in Step 1 the set of equations is solved. Under criterion (1) v_s denotes the total expected discounted reward of a process starting in state s and running over an infinite number of stages. Under criterion (3) v_s is the relative value compared to state $\hat{s}$. The difference between the relative value of two states denotes the amount we are willing to pay for stating in the state with the highest relative value. In Step 2 we update the current policy. This is repeated until a better policy can not be found (Step 3). Finally, observe that if the time

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Step 0: Choose a policy δ.
Step 1: Solve the set of linear equations in Table 2.
Step 2: For each state s, find the action a that maximizes the expression given in Table 2, and
  set δ'(s) = a.
Step 3: If δ' = δ then stop; otherwise go to Step 1.
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Figure 2: Policy iteration algorithm for an infinite-horizon ordinary MDP.

Table 2: Equations and expressions to be used in the policy iteration algorithm.

	Step 1		Step 2
Criterion	Equations	Unknowns	Expression
(1)	$v_s = r_s^\delta + \sum_{\hat{s}\in S} \beta_s^\delta p_{s\hat{s}}^\delta v_{\hat{s}}, \forall s \in S$	$v_1, \dots, v_{\lvert S \rvert}$	$r_s^\delta + \sum_{\hat{s}\in S} \beta_s^\delta p_{s\hat{s}}^\delta v_{\hat{s}}^\delta$
(3)	$v_s = r_s^\delta - g t_s^\delta + \sum_{\hat{s}\in S} p_{s\hat{s}}^\delta v_{\hat{s}}, \forall s \in S, v_{\hat{s}} = 0$	$v_1, \dots, v_{\lvert S \rvert}, g$	$r_s^\delta - t_s^\delta g(\delta) + \sum_{\hat{s}\in S} p_{s\hat{s}}^\delta v_{\hat{s}}^\delta$

between each decision epoch is constant ($t_s^a = 1$ and $\beta_s^a = \beta$), then the recursive formulas in Table 2 reduce to the well-known formulas for an ordinary MDP. For more advanced variants of the policy iteration algorithm see Puterman (1994).

2.3 Hierarchical MDPs

Hierarchical MDPs are an attempt to decompose the state space and reduce the number of states in the MDP. The approach also provide a more intuitively way of modeling the stochastic process. Moreover, it reduces the number of equations which must be solved simultaneously under policy iteration. We consider hierarchical MDPs with multiple levels also referred to as multi-level hierarchic Markov processes. A hierarchical MDP is an infinite stage MDP with parameters defined in a special way, but nevertheless in accordance with all usual rules and conditions relating to such processes. The basic idea of the hierarchic structure is that stages of the process can be expanded to a so-called *child processes* which again may expand stages further to new child processes leading to multiple levels.

A stage in a process with three levels is illustrated in Figure 3. The infinite horizon process at level 0 is named the *founder process* and is the only process in the structure which is not the child of a parent process. Each node corresponds to a state at different levels and stages. A child process (oval box) is a finite horizon MDP and is uniquely defined by a given stage, state, and action of its parent process (the specific link/edge from the parent to the child). For each finite horizon process an initial probability distribution of the states at stage 1 is assumed, i.e., a fictitious stage 0 with only one state and one action is added to the model. As a result given a state and action at the parent level a transition to the child process can be represented deterministically (edges in Figure 3). Moreover, a set of terminal probabilities are given representing the transition probabilities back to the parent process when the last stage ends (the links from the last stage in the child in Figure 3).

Note that a finite horizon process at level $l > 0$ is uniquely defined by a sequence of stages,

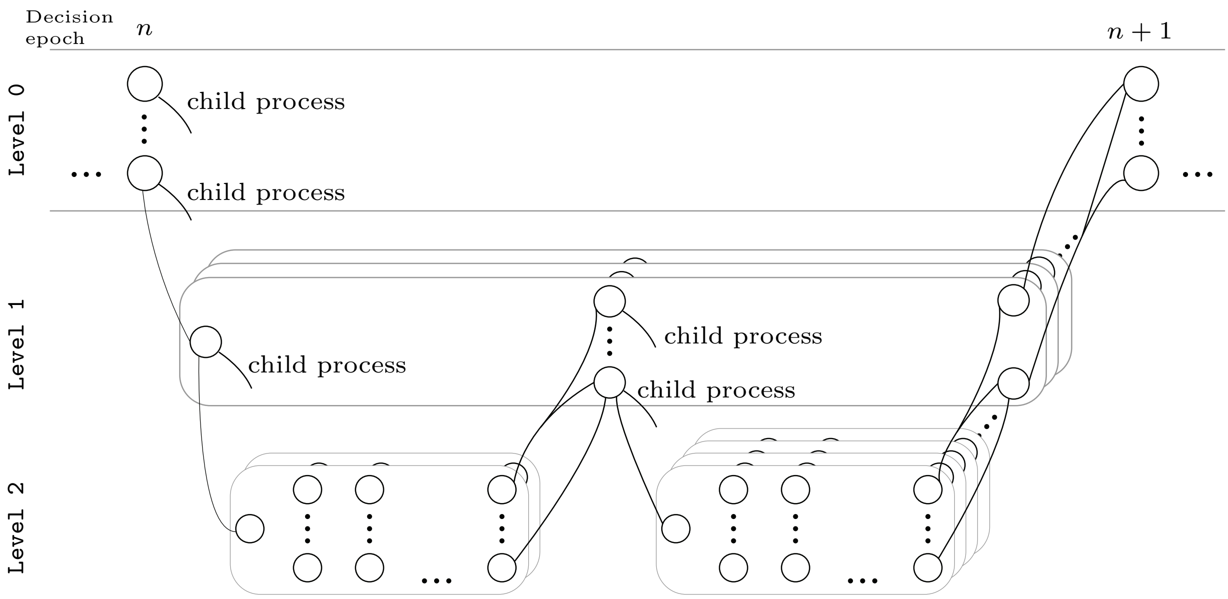

Figure 3: Illustration of a stage in a hierarchial MDP. Level 0 indicates the founder level, and the nodes indicate states at the different levels and stages. A child process (oval box) represents a finite horizon MDP and is uniquely defined by a given state and action of its parent process (the specific link/edge from the parent to the child). Links at the last stage of a process illustrate the possible transitions back to the parent process when the child process ends.

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Step 0: Set $v_s(0)=0, \forall s \in S_{\rho_0}$ and $g=0$ (under criterion (3)). Perform an expanded value iteration to
  find the expanded policy $\delta_{\rho_0}$ and parameters $r_s^{\delta_{\rho_0}}$, $t_s^{\delta_{\rho_0}}$, $\beta_s^{\delta_{\rho_0}}$ and $p_{s\hat{s}}^{\delta_{\rho_0}}$.
Step 1: Solve the set of linear equations in Table 2 using the parameters of the founder
  process.
Step 2: Perform expanded value iteration to find the expanded policy $\delta'_{\rho_0}$ and parameters $r_s^{\delta'_{\rho_0}}$, $t_s^{\delta'_{\rho_0}}$
  , $\beta_s^{\delta'_{\rho_0}}$ and $p_{s\hat{s}}^{\delta'_{\rho_0}}$.
Step 3: If $\delta'_{\rho_0}=\delta_{\rho_0}$ then stop; otherwise redefine $\delta_{\rho_0}$ to the new policy and go to Step 1.
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Figure 4: Hierarchical policy iteration algorithm for an hierarchical MDP.

states and actions $\rho = (s_0, a_0, n_1, s_1, a_1, \ldots, n_{l-1}, s_{l-1}, a_{l-1})$ and at level 0 we only have the infinite horizon founder process which we will denote ρ_0. We will use the notation in Section 2.1 and Section 2.2 given a specific process ρ; however, an action a is not nessasarely identical to an action as it is usually defined in an MDP. In addition to the selection of a specific process we also have to choose which policy to follow during its child processes.

Let δ_ρ denote an *expanded policy* of process ρ, i.e., a function that assigns to each state s a fixed action $a = \delta_\rho(s)$, i.e., an expanded policy provides the decision maker with a plan of which action to take given stage and state in the parent process and all its child processes. Then the reward $r_s^{\delta_\rho}(n)$, expected length $t_s^{\delta_\rho}(n)$, discount factor $\beta_s^{\delta_\rho}(n)$, and transition probabilities $p_{s\hat{s}}^{\delta_\rho}(n)$ can be calculated recursively by processing the child processes from the lowest levels and upward toward the parent process ρ. Hence an *expanded value iteration* can be applied. Under the *total expected discounted reward* criterion (1) and given a set of terminal rewards, the optimal policy δ_ρ of a finite horizon process can be found by recursively applying value iteration (2) from the lowest levels and upward toward the parent process ρ. The same holds when considering the *average reward per time unit* criterion (3) where we must solve the following recursive equations:

$$v_n(s) = \begin{cases} \max_{a \in A_{s,n}} \left\{ r_s^a(n) - g t_s^a + \sum_{\hat{s} \in S_{n+1}} p_{s\hat{s}}^a(n) v_{n+1}(\hat{s}) \right\} & n < N \\ r_s^{a_N}(N) & n = N \end{cases},$$

Note that an additional average reward g must be chosen together with the terminal values. For further details see Kristensen and Jørgensen (2000).

We can also apply a single iteration of expanded value iteration to the founder process to determine all the parameters needed to solve the set of equations when considering policy iteration. A hierarchical policy iteration algorithm can now be formulated in Figure 4. It combines policy iteration at the founder level and value iteration at the other levels. First some initial values are chosen in Step 0 and the expanded policy and the parameters of the founder process are calculated. Next the linear equations at the founder level are solved in Step 1 and used as terminal values in the expanded value iteration in Step 2. If no new policy is found the algorithm stops in Step 3.

3 MDP models applied to cattle farming

This section gives an overview of MDPs applied to cattle farming problems. Around 60 papers describing more than 40 different models were found in this area. Table 3 summarizes the models by listing their structure in terms of the number of levels (the value 1 indicates an ordinary MDP), the criterion of optimality, the state variables with number of levels/classes, stage lengths with maximum number of stages, decisions being optimized, application area, and supplementary information. Each row in the table corresponds to a model and reference to the paper(s) describing it is given in the first column. It should be noticed that it is not always clear whether a paper should be classified as describing a new model (by further developing an existing model) or it should be classified as just an application of an existing model.

Only *decision* models are included in the survey. Simple Markov chain models are not mentioned even though they are, of course, closely related to MDPs since an MDP with a predefined policy is a Markov chain. Examples of such, not included, Markov chain models are Allore et al (1998); Cabrera (2012); Giordano et al (2012); Noordegraaf et al (1998), as well as Jalvingh et al (1993a,b, 1994).

Many of the models mentioned in the survey are by the authors themselves presented as *dynamic programming* models and the term Markov decision process is seldom mentioned. Dynamic programming exists in a deterministic version and a stochastic version, and particularly the stochastic version is identical to the MDP concept described in this chapter. Very often, however, the use of the term dynamic programming implies that the optimization method is value iteration. The deterministic version is also compatible with an MDP, but such models are degenerate in the sense that for any stage n, state s, and action a there exists a state s' at stage $n+1$ where $p^a_{ss'} = 1$. Accordingly, we have for any state $\hat{s} \neq s'$ that $p^a_{s\hat{s}} = 0$.

In a book Kennedy (1986) reviewed dynamic programming applications to agriculture until the early eighties. As a main rule, models mentioned in that book are omitted, but for the most important application area, which is dairy cow replacement, also models mentioned by Kennedy (1986) are included. The main reason is that the study by Giaever (1966) is so important that it would be preposterous to omit it.

The vast majority of papers and models address problems related to dairy cows. A few models consider growing cattle (the review by Kennedy, 1986, contains several very early applications to growing cattle). Nielsen et al (2004) and Nielsen and Kristensen (2007) consider the raising of steers and Pihamaa and Pietola (2002) study the effect of beef cattle management under agricultural policy reforms in Finland. Also management of heifers (Mourits et al, 1999a,b) has been studied. All models are defined at the individual animal level and since all of them also basically consider the replacement problem, they reflect a chain of animals successively replacing each others over a finite or infinite time horizon. They therefore all have the action “Replace” as an option. The alternative to replacement is, of course, to keep the animal, and many models only have “Keep” as an alternative to “Replace”. Many models describing cows and heifers also have an “Inseminate” action, and the models optimizing raising of steers and heifers have actions defining the feeding level in some sense.

The first models published until the mid-eighties were ordinary MDPs solved by value iteration over a number of stages typically aiming at approximating an infinite horizon. The criterion of optimality was typically maximization of expected discounted reward, which is still today the most commonly used criterion. The concept of hierarchical MDPs was described by Kristensen (1988), and over the following years it has been increasingly used in cattle models. In total, 11 of the models mentioned in Table 3 are hierarchical. Most of the recent hierarchical models have been implemented in the MLHMP software system developed by Kristensen (2003). The technique has made it possible to handle even very large models with millions of states like Demeter et al (2011), Nielsen et al (2010), and Houben et al (1994). The introduction of hierarchical models also implies that policy iteration has become a common optimization technique (for the founder process).

When it comes to state variables, the models include age of the animal as a state variable. For dairy cows it is typically measured by lactation number and often also stage of lactation. Also the reproductive state (typically measured by month of conception or length of calving interval) and the milk yield level are usually included in the dairy cow models. In the beginning the health status was not included in the models, but starting with Stott and Kennedy (1993), Kennedy and Stott (1993), and Houben et al (1994) mastitis has often been included in the state space. In recent years (Bar et al, 2008a,b; Cha et al, 2011; Heikkila et al, 2012) mastitis has been studied intensively. Also other diseases have occasionally been included (Cha et al, 2010; Grohn et al, 2003; Heikkila et al, 2008).

When comparing state variables across models it is important to remember that in hierarchical models some of the state variables are typically implicitly given by stage number. This is typically the case for properties like age (lactation number and lactation stage for dairy cows) and/or season. Thus, in hierarchical models it is most often not necessary to include state variables for such properties because they are given by the model structure. Hence, the same problem formulated as a hierarchical model will typically have fewer state variables than if it had been formulated as an ordinary MDP.

Stage lengths (for hierarchical models at the most detailed level) vary from one day as in Kalantari and Cabrera (2012); Nielsen et al (2010) to typically a lactation period in many early models. Geographically, the largest number of models (12) describe US conditions, but also models for UK conditions (8), Dutch conditions (6), Danish (4) and Finish conditions (4) are common. Two models describe MDPs developed for New Zealand, two for Ireland, two for Canada, and for each of the countries Iran, Costa Rica, France, and Israel one model has been developed.

Very few papers actively discuss how to satisfy the Markov property, but in many papers it is obvious that the problem is considered (in other papers it is ignored). The preferred method for (approximate) fulfilment of the Markov property has been by use of memory variables where milk yield of previous lactation is remembered. This tradition goes back to van Arendonk (1985b) and has been continued in many subsequent models using that model as a basis (see the "Misc" column of Table 3). The same approach was used by Kristensen (1987, 1989). The main drawback of memory variables is that they contribute considerably to the curse of dimensionality. This was realized already by Giaever (1966) who instead defined milk yield as a weighted index of all lactations until now. He showed how it was possible to define the weight coefficients of the index in such a way that the Markov property was not violated. Also McArthur (1973) defined an index which in his case was a simple average of lactation yields. Thus, the state space was reduced, but the Markov property was not satisfied.

Another approach used in several models is to express the milk yield as partly resulting from a permanent property of the cow. This approach was used by Kristensen (1987, 1989) (as a supplement to the memory variable also included). In the models developed at Cornell University (Bar et al, 2008a,b; Cha et al, 2010, 2011) the permanent property was the only approach used to satisfy the Markov property. All the models mentioned are hierarchical MDPs which are particularly well suited for handling permanent traits. Nevertheless, Harris (1990) seems to have used a similar principle in an ordinary MDP.

When the principles of Bayesian updating was described by Kristensen (1993) and (independently) applied by Kennedy and Stott (1993) a new tool became available for model builders. Instead of memory variables, the Bayesian updating focuses on estimating an abstract latent milk yield capacity of a cow based on *all* observed milk yield records. It was, however, not until the models by Nielsen et al (2010) and Demeter et al (2011) that it was implemented as a main feature. In other application areas (Kristensen and Søllested, 2004a,b; Lien et al, 2003; Verstegen et al, 1998) it was used earlier.

Table 3: Overview over literature using MDPs for modeling within cattle farming.

Paper[a]	Levels[b]	Criterion[c]	State variables[d]	Stage Length[e]	Decisions[f]	Application[g]	Misc
Kalantari and Cabrera (2012)	1	*DR* (VI)	lactation (9), days in preganacy (282), DIM (750), milk yield (5)	day (∞)	K, R	dairy (US)	Study the effect of reproductive performance.
Heikkila et al (2012)	1	*DR* (PI)	month (78), culling reason (3), mastitis cases (5)	month (∞)	K, R	dairy (FIN)	Focus on clinical mastitis
Langford and Stott (2012)	1	*DR* (VI)	parity (12), milk yield level (15)	parity (20)	K, R	dairy (UK)	Extension of Stott (1994) which study the effect on welfare
Cha et al (2011)	3	*DR* (HPI)	permanent milk yield level (5); dummy (1); temporary milk yield level (5), pregnancy state (9), clinical mastitis state (13)	cow life (∞); parity (8); month (20)	I, K, R	dairy (US)	Lactation number and stage of lactation known from stage number. Extension of the work by Bar et al (2008b) and Cha et al (2010).
Demeter et al (2011)	4	*DR* (HPI)	permanent milk yield potential (PMYP) estimated at first calving(13); PMYP estimated at the beginning of lactation (13), months open previous lactation (8); PMYP estimated this month (13), temporary milk yield capacity (13), pregnancy state (2); PMYP estimated this month (13), temporary milk yield capacity (13)	cow life (∞); parity (12); month/gestation period (18); month (9)	I, K, R	dairy (NL)	Used to assess herd level implication of genetic selection strategies. Lactation number, stage of lactation and month of pregnancy known from stage numbers.
Cabrera (2010)	1	R/T (LP)	parity (15), month in lactation (24), pregnancy status (10)	month (∞)	K, R	dairy (US)	Consider different diets and nitrogen excretion
Cha et al (2010)	3	*DR* (HPI)	permanent milk yield level (5); dummy (1); temporary milk yield level (5), pregnancy state (9), lameness state (13)	cow life (∞); parity (8); month (20)	I, K, R	dairy (US)	Lactation number and stage of lactation known from stage number. Extension of the work by Bar et al (2008b) with focus on lameness.
Kalantari et al (2010)	1	DR (VI)	lactation (12), month after calving (24), milk production class (15), pregnancy status (10)	lactation (180)	K, R	dairy (IR)	A modification of van Arendonk and Dijkhuizen (1985) applied to Iran conditions.
Nielsen et al (2010)	3	*DR* (HPI)	dummy (1); milk yield potential (MYP) estimated at the beginning of lactation (13); combination of MYP estimated until present day and temporary milk yield level (45 combinations), drying off week (32)	cow life (∞); parity (10); day (483)	K, R	dairy (DK)	Lactation number and stage of lactation known from stage number. Focus on management. Bayesian updating used.
Bar et al (2008a,b)	3	*DR* (HPI)	permanent milk yield level (5); mastitis in previous lactation (2); temporary milk yield level (5), pregnancy state (9), mastitis state in present lactation (13)	cow life (∞); parity (8); month (20)	I, K, R	dairy (US)	Lactation number and stage of lactation known from stage number. Focus on cost of clinical mastitis.

(*Continued on next page*)

Table 3: Overview over literature using MDPs for modeling (cattle farming - table continued).

Paper[a]	Levels[b]	Criterion[c]	State variables[d]	Stage Length[e]	Decisions[f]	Application[g]	Misc
Heikkila et al (2008)	1	*DR* (PI)	lactation (10), milk yield (3), health status (3)	lactation (∞)	K, R	dairy (FIN)	Focus on diseases and milk yield.
Nielsen and Kristensen (2007); Nielsen et al (2004)	4	R/T (HPI), R/Q (HPI)	birth month (12); live weight (up to 26) previous winter feeding level (2), weigh gain (5); weight gain at fattening (3)	steer life (∞); seasons (summer/winter) (6); month (up to 6); month (4)	G, Fe, Fa, R	steer (DK)	Nielsen et al (2004) consider average reward per steer while in Nielsen and Kristensen (2007) the average reward per time unit is maximized
de Vries (2006)	1	*DR* (VI)	lactation (12), days open (10), month of lactation (24), milk yield (15)	month (∞)	K, R	dairy (US)	Extension of model by de Vries (2004).
Stott et al (2005)	1	*DR* (VI)	lactation (12), milk yield (15)	lactation (20)	K, R	dairy (UK)	Studies financial incentive to control paratuberculosis. Extension of model by Stott (1994).
de Vries (2004)	1	*DR* (VI)	lactation (12), days open (10), month of lactation (24), milk yield (15), month of calving (12)	month (∞)	K, R	dairy (US)	Studies the effect of delayed replacement with seasonal cow performance.
Grohn et al (2003)	1	*DR* (VI)	lactation (12), days open (10), month of lactation (20), milk yield (5), month of calving (12), disease state (240)	month (60)	I, K, R	dairy (US)	Extension of models by Delorenzo et al (1992) and Mccullough and Delorenzo (1996b).
Stott et al (2002)	1	*DR* (VI)	lactation (12), milk yield (15), somatic cell count (11)	lactation (20)	K, R	dairy (UK)	Extension of model by Stott (1994).
Pihamaa and Pietola (2002)	1	*DR* (VI)	live weight (507)	week (326)	Fe, K, R	beef (FIN)	Study the effect of agricultural policy reforms in Finland.
Rajala-Schultz and Grohn (2001)	1	*DR* (VI)	lactation (12), production level (5), month of calving (12), month of lactation (19), days open (10)	month (60)	I, K, R	dairy (FIN)	Compares optimal decisions with farmer decisions. Use of model by Mccullough and Delorenzo (1996b).
Vargas et al (2001)	1	*DR* (VI)	lactation l (12), stage in lactation (11), milk yield l (15), $l-1$ (15)	month (180)	I, K, R	dairy (CR)	Based on model by van Arendonk and Dijkhuizen (1985).
Rajala-Schultz et al (2000a,b)	1	*DR* (VI)	parity (12), days open (10), stage of lactation (19), production level (3, 5, 7), month of calving (12)	month (48-120)	I, K, R	dairy (FIN)	Use of model by Mccullough and Delorenzo (1996b).
Yalcin and Stott (2000)	1	*DR* (VI)	lactation (12), milk yield (15), somatic cell count (11)	lactation (20)	K, R	dairy (UK)	Extension of work by Stott (1994)
Cardoso et al (1999a,b)	1	*DR* (VI)	lactation l (12), stage in lactation (11), milk yield l (15), $l-1$ (15)	month (240)	I, K, R	dairy (BR)	Use of model by van Arendonk and Dijkhuizen (1985).
Mourits et al (1999a,b)	2	*DR* (HPI)	month of birth (12); body weight (173), reproductive state (32), prepubertal growth rate (3)	rearing period (∞); month (30)	Fe, I, K, R	heifers (NL)	Age of heifer known from stage number. The keep and inseminate decisions can be done under different growth strategies

(*Continued on next page*)

Table 3: Overview over literature using MDPs for modeling (cattle farming - table continued).

Paper[a]	Levels[b]	Criterion[c]	State variables[d]	Stage Length[e]	Decisions[f]	Application[g]	Misc
Yates and Rehman (1998)	1	*DR* (LP)	lactation (12), genetic level (4)	year (10)	K, R	dairy (UK)	The keep decision has 2 options: produce calf for replacement or for beef.
Dekkers et al (1998)	1	*DR* (VI)	lactation l (12), month in lactation (16), milk yield l (15), calving intervals (6)	month (180)	I, K, R	dairy (CDN)	Quantify the impact of persistency of lactation. Adaptation of the work in van Arendonk and Dijkhuizen (1985)
Haran (1997)	2	*DR* (HPI)	month of first calving (12); current month (12), milk production level (15), time of conception (5)	cow life (∞); lactation stage (72)	I, K, R	dairy (IRL)	Lactation number and stage of lactation known from stage number.
Mccullough and Delorenzo (1996a,b)	1	*DR* (VI)	lactation (12), production level (15), month of calving (12), month of lactation (19), days open (10)	month (60)	I, K, R	dairy (US)	Focus: levels of state variables, milk price and management inputs. Model based on Delorenzo et al (1992)
Houben et al (1994)	2	*R/T* (HPI)	dummy (1); milk production l (15), $l-1$ (15), calving interval (18), mastitis current month (2), mastitis cases l (4), $l+1$ (4)	life span of a cow (∞); month (204)	I, K, R	dairy (NL)	Focus on mastitis
Stott (1994)	1	*DR* (VI)	lactation (12), yield class (15)	lactation (∞)	K, R	dairy (UK)	Uses bayesian updating for milk yield
Kennedy and Stott (1993)	1	*DR* (VI)	lactation l (12), yield class (5), mastitis status $l-1$ (2)	lactation (∞)	K, R	dairy (UK)	Focus: model and bayesian updating
Stott and Kennedy (1993)	1	*DR* (VI)	lactation number (12), mastitis state (2)	lactation (∞)	K, R	dairy (UK)	Focus on clinical mastitis.
Delorenzo et al (1992)	1	*DR* (VI)	lactation (12), production level (15), month of calving (12), month of lactation (16), days open (7)	month (240)	I, K, R	dairy (US)	Model based on van Arendonk (1986)
Dekkers (1991)	1	*DR* (VI)	lactation l (12), stage in lactation (11), milk yield l (15), $l-1$ (15), time of conception (6)	month (180)	I, K, R	dairy (CDN)	Studies economic values for breeding goals. Adaptation of the work in van Arendonk and Dijkhuizen (1985)
Boichard (1990)	1	*DR* (VI)	lactation l (6), lactation stage (22), stage of conception (7), calving date (18), milk yield in l (9), $l-1$ (9)	20 days (200)	I, K, R	dairy (F)	Focus: economic value of conception
Harris (1990)	1	*DR* (VI)	lactation (10), best linear prediction of future milkfat production, milk volume production, milk protein production, breed, calving date (6)	year (20)	K, R	dairy (NZ)	It is not clear from the description whether an optimization is performed or the model is only used for simulation.

(*Continued on next page*)

Table 3: Overview over literature using MDPs for modeling (cattle farming - table continued).

Paper[a]	Levels[b]	Criterion[c]	State variables[d]	Stage Length[e]	Decisions[f]	Application[g]	Misc
Kristensen (1989); Kristensen and Thysen (1991a,b)	2	R/Q (HPI)	estimated genetic class at first calving (5); milk yield of present lactation (15), milk yield of previous lactation (15), length of calving interval (8)	cow life (∞); 4 week period (108)	K, R	dairy (DK)	Lactation number and stage of lactation known from stage number. Average reward per kg milk is maximized. Extension of work by Kristensen (1987). The model is later applied by Kristensen and Thysen (1991a,b)
Rogers et al (1988a,b)	1	*DR* (VI)	lactation l (12), stage in lactation (11), milk yield l (15), $l-1$ (15), time of conception (6)	month (180)	I, K, R	dairy (US)	Adaptation of the work in van Arendonk and Dijkhuizen (1985)
Kristensen (1987)	2	*DR* (HPI)	estimated genetic class at first calving (5); milk yield of present lactation (15), milk yield of previous lactation (15), length of calving interval (8)	cow life (∞); lactation stage (24)	K, R	dairy (DK)	Lactation number and stage of lactation known from stage number.
van Arendonk (1986)	1	*DR* (VI)	lactation l (12), stage in lactation (11), milk yield l (15), time of conception (6), month of calving (12)	month (180)	I, K, R	dairy (NL)	Extension of the work in van Arendonk (1985b)
van Arendonk (1988); van Arendonk and Dijkhuizen (1985)	1	*DR* (VI)	lactation l (12), stage in lactation (11), milk yield l (15), $l-1$ (15), time of conception (6)	month (180)	I, K, R	dairy (NL)	Extension of the work in van Arendonk (1985b)
van Arendonk (1985a,b)	1	*DR* (VI)	lactation l (12), stage in lactation (11), milk yield l (15), $l-1$ (15)	month (240)	I, K, R	dairy (NL)	The model has had a huge impact on later models.
Ben-Ari and Gal (1986); Ben-Ari et al (1983)	1	DR (VI)	lactation, milk yield, body weight	lactation (∞)	K, R	dairy (IL)	Ben-Ari and Gal (1986) consider how to solve the multi-component system.
Killen and Kearney (1978)	1	*R* (VI)	lactation number (9)	lactation (20)	R, K	dairy (IRL)	Very small model.
Stewart et al (1977, 1978)	1	*DR* (VI)	lactation (7), body weight (5), 305d milk yield (11), milk fat pct (7)	lactation (10)	R, K	dairy (CDN)	Stewart et al (1977) describe the model and Stewart et al (1978) consider different breeds. Culling decisions were assumed to occur at 60 days postcalving
McArthur (1973)	1	*R* (VI)	lactation number (7), milk production level (80)	lactation (15)	K, R	dairy (NZ)	Milk yield represented as average over lactations.
Smith (1971, 1973)	1	*DR* (VI)	lactation l (6), yield in l (29), $l-1$ (29), calving interval (3)	lactation (15)	R, K	dairy (US)	Far more detailed model than the one by Giaever (1966).
Giaever (1966)	1	*DR* (VI)	lactation number (5), calving interval (3), milk yield (7)			dairy (US)	Alternative optimization methods described. Important considerations about Markov property.

[a] Papers have been ordered in reverse order of year.
[b] Number of levels in the MDP. If 1 then the MDP is an ordinary MDP.
[c] DR = expected discounted reward, R = expected reward, R/T = average reward per time unit, R/Q average reward per quantity unit. Algorithm used is given in parentheses (VI = value

iteration, PI = policy iteration, HPI = hierarchical policy iteration, LP = Linear programming).

[d] State variables for each level in the process (separated with semicolon). The number of levels/classes of each state variable is given in parentheses.

[e] Stage length at each level in the process (separated with semicolon). Maximum number of stages given in parentheses.

[f] R = replace, K = keep, I = Inseminate, G = Grazing, Fe = Feeding intensity, Fa = Fattening.

[g] Animal group applied to. The country from which the parameters has been estimated is given in parentheses.

4 MDP models applied to pig farming

Table 4 summarizes MDPs applied to pig farming along the same guidelines as for the cattle applications in Table 3. A total of 17 papers describing 12 different models were identified. As with the cattle models only decision models are included implying that simple Markov chain models are excluded. Examples of such not included Markov chain models are Jalvingh et al (1992a,b) and Pla et al (2003).

Analogously to the many dairy cow replacement models in the previous section a total of 6 sow replacement models were found. The remaining papers (6) address problems related to production of finishers. Also the pig models are in some sense replacement models, but unlike the cattle models there are also examples of MDPs defined at group level. Thus, Kristensen et al (2012) model a pen, and Toft et al (2005) as well as Kure (1997a,b,c) model a batch of finishers. There are, however, also examples of finisher models (Glenn, 1983; Jørgensen, 1993; Niemi, 2006) defined at individual animal level. The sow models are all defined at individual animal level.

Decisions considered in the sow models are in addition to “Keep” and “Replace” also insemination method and number of inseminations to accept before culling for infertility. In finisher models decisions are the marketing policy and, some times, the feeding level. As concerns the optimization method the first models published were ordinary MDPs based on value iteration optimizing expected reward or expected discounted reward. Later hierarchical models became the norm with the deterministic model by Niemi (2006) as an exception. Also for the hierarchical pig models the preferred software tool has been the MLHMP system described by Kristensen (2003).

In all models the age of the animal(s) is included either as a state variable or indirectly through the stage number in hierarchical models. In the sow models litter size is often included either directly or through Bayesian updating of a latent litter size potential as in Kristensen and Søllested (2004a,b) and Rodriguez et al (2011). Also, the number of unsuccessful inseminations is sometimes directly or indirectly (through the model structure) taken into account. One model by Rodriguez et al (2011) included a weak sow index defined by clinical observations in the state space.

Stage lengths vary from one day as in Niemi (2006) to a reproduction period (parity) in several models. Geographically, the largest number of models (7) describe Danish conditions, but also models for Dutch, UK, Spanish, and Finish conditions are found.

As concerns the Markov property, the approach has been the same as with dairy models. Dutch models (Huirne and Hardaker, 1998; Huirne et al, 1988, 1991, 1993) used memory variables (2 or 3 previous litter sizes). Later models (Jørgensen, 1992; Kristensen and Søllested, 2004a,b; Kristensen et al, 2012; Rodriguez et al, 2011) used Bayesian updating.

Table 4: Overview over literature using MDPs for modeling pig farming.

Paper[a]	Levels[b]	Criterion[c]	State variables[d]	Stage Length[e]	Decisions[f]	Application[g]	Misc
Kristensen et al (2012)	2	R/T (HPI)	dummy (1); number of pigs remaining (21), estimated permanent growth potential (7), estimated temporary growth potential (7), estimated within pen standard deviation (9)	prod. cycle in pen (∞); week (17)	D_δ	finishers (DK)	Embeds a Dynamic Linear Model linking automatically recorded live weights to state variables. Group level: models a pen.
Rodriguez et al (2011)	3	R/T (HPI)	dummy (1); exp. serially correlated effect (21), exp. permanent litter size potential (21)[h]; health status (2), gestation status (3), litter size (21), weak sow index of previous parity (5), weak sow index of present parity (5)	sow life (∞); parity (12); parity phases (3)	NM, AI, R, K, M_i	sows (DK)	Extension of work by Kristensen and Søllested (2004a,b). The weak sow index is based on clinical observations.
Toft et al (2005)	2	? (HPI)	disease transition (5); configurations of susceptible and infectious pigs (?), fraction of pigs still present (5)	prod. cycle in pen (∞); day and week (88)	V, T, D_π, K	finishers (DK)	Group level: models a batch.
Niemi (2006)	1	*DR* (VI)	lean tissue weight (37), fat tissue weight (52)	day (1800)	R, P, E	finishers (FIN)	Deterministic model. Very detailed control options.
Kristensen and Søllested (2004a,b)	3	R/T	dummy (1); exp. serially correlated effect (21), exp. permanent litter size potential (21)[h]; health status (2), gestation status (3), litter size (21)	sow life (∞); parity (12); parity phases (3)	NI, AI, R, K, M_i	sows (DK)	Uses Bayesian updating to estimate litter size
Pla et al (2004)	1	R/T (PI)	reproductive state (9), parity (11)	variable (from event to event)	R, K	sows (E)	Uses herd data for estimation of transition probabilities.
Kure (1997a,b,c)	2	*DR* (HPI)	observed live weigh, observed carcass leanness	prod. cycle in pen (∞); weeks of delivery (4)	D_δ, E	finishers (DK)	Uses Recursive Dynamic Programming in child process. Group level: models a batch.
Jørgensen (1993)	2	*DR* (VI)	dummy (1); weeks since start (5), pigs in pen (32) [161]	prod. cycle in pen (∞); week (5)	D_δ, E	finishers (DK)	The first period at second level is actually 10 weeks (minimum feeding time)
Jørgensen (1992)	2	L/T (VI)	dummy (1); parity (20), exp. random effect and influence on litter size (100) [2001]	sow life (∞); parity (20)	R, K	sows (DK)	Litter size based on bayesian updating. Hierarchical structure imply reduced state space compared to (Huirne et al, 1993)
Huirne and Hardaker (1998); Huirne et al (1991, 1993)	1	*DR* (VI)	parity p (11), litter size in $p-1$, $p-2$ (12), unsuccessful breedings in p (4) [5633]	parity (70)	R, K	sows (NL)	Huirne and Hardaker (1998) uses the MDP as a sub-model.
Huirne et al (1988)	1	*DR* (VI)	parity p (15), litter size in $p-1$, $p-2$, $p-3$ (20)	parity (50)	R, K	sows (NL)	Litter size based on a dynamic formula
Glenn (1983)	1	*R* (VI)	live weight (80), carcass composition ()	5 days (17)	G, P	finishers (UK)	Deterministic model

[a] Papers have been ordered in reverse order of year.
[b] Number of levels in the MDP. If 1 then the MDP is an ordinary MDP.
[c] *DR* = discounted reward, L/T = avg. litter size per time unit, R/T = avg. reward per time unit. Algorithm used is given in parentheses (VI = value iteration, PI = policy iteration, HPI =

hierarchical policy iteration).

[d] State variables for each level in the process (separated with semicolon). The number of levels/classes of each state variable is given in parentheses.

[e] Stage length at each level (separated with semicolon). Maximum number of stages given in parentheses.

[f] V = vaccinate, T = treat, D_π = deliver π pigs, R = replace, K = keep, D_δ = deliver pigs with weight above δ, E = empty the pen, NM = natural mating, AI = Artificial Insemination, M_i = allow i matings, P = protein level, E = energy level, G = gain.

[g] Animal group applied to. The country from which the parameters has been estimated is given in parentheses.

[h] A dummy state representing the pig has been culled is also included in the model.

5 MDP models applied to other areas

Even though most models have been developed for applications within cattle and pig production, a few papers within other applications exist in the literature.

Table 5 summarizes MDPs applied to other areas within livestock farming along the same guidelines as in the tables for cattle and pig applications (Table 3 and 4). A total of 5 papers were identified. In addition to those listed in the table, Kennedy (1986) reviews a number of very early applications to laying hens, broilers and sheep.

Verstegen et al (1998) used an MDP as a tool for comparing different management information systems performance against the optimal decisions found by the MDP and van Asseldonk et al (1999) used an MDP to optimize which IT solutions to implement on farm. The remaining papers focus on food and mouth disease (FMD) (Ge et al, 2010a,b) and how to compute an adaptive control strategy of an animal disease among a set of farms (Viet et al, 2012). Decisions considered in the models are "Keep", "Replace", if the farm should investment in a certain IT solution, vaccination strategy, and different FMD control options.

Due to the various applications state variables differ much. Examples are IT investment status, epidemic situation, infected, and month etc. Stage lengths vary from one day as in Ge et al (2010a) to a year (van Asseldonk et al, 1999). Two papers use ordinary MDPs based on value iteration optimizing expected discounted reward and three papers use hierarchical models, with two implemented using the MLHMP software (Kristensen, 2003).

The models by Ge et al (2010a) and Ge et al (2010b) use Bayesian updating to estimate the disease spread properties of the FMD virus causing the FMD outbreak, and Verstegen et al (1998) use Bayesian updating to estimate the properties of hypothetical projects.

Table 5: Overview over literature using MDPs within other areas than cattle and pig farming.

Paper[a]	Levels[b]	Criterion[c]	State variables[d]	Stage Length[e]	Decisions[f]	Application[g]	Misc
Viet et al (2012)	1	*DR* (VI)	current month (12), S = susceptible $(N+1)$, I = infected $(N+1)$, V = vaccinated $(N+1)$; only state combinations where $S+I+V=N$	1 month (∞)	V, no V	disease control (F)	Total number of herds N varied from 50 to 400.
Ge et al (2010b)	2	*DR* (HPI)	epidemic situation (3), export ban (2), infection index (5), estimated growth potential of epidemic (5), uncertainty of growth potential (5)	duration of epidemic (∞); 10 day periods (10)	BP, V, PC	FMD control (NL)	Modification of work by Ge et al (2010a).
Ge et al (2010a)	3	(HPI)	epidemic situation (2), infection index (5), estimated growth potential of epidemic (5), uncertainty of growth potential (5)	duration of epidemic (∞); 10 day periods (10); 1 day (10)	SP, BP, V, PC, STOP	FMD control (NL)	See also Ge et al (2010b).
van Asseldonk et al (1999)	1	*DR* (VI)	IT investment status (11^5): automatic concentrate feeder (11), activity measurement (11), milk production measurement (11), milk temperature measurement (11), conductivity measurement (11)	year (20)	Invest	IT investment (NL)	Studies investments in IT equipment at farm level. Deterministic model.
Verstegen et al (1998)	2	? (HPI)	number of production weeks, yield per production week	project life (∞); year (10)	R, K	MIS[i] evaluation (NL)	Project age known from stage number. Model with Bayesian updating. Use the MDP as a tool for comparing against farmers choice.

[a] Papers have been ordered in reverse order of year.
[b] Number of levels in the MDP. If 1 then the MDP is an ordinary MDP.
[c] *DR* = discounted reward, L/T = litter size per time unit, R/T = reward per time unit. Algorithm used is given in parentheses (VI = value iteration, PI = policy iteration, HPI = hierarchical policy iteration).
[d] State variables for each level in the process (separated with semicolon). The number of levels/classes of each state variable is given in parentheses.
[e] Stage length at each level (separated with semicolon). Maximum number of stages given in parentheses.
[f] V = vaccinate, BP = Basic control (FMD), PC = Preemptive culling (FMD), SP = Stop program (FMD), R = replace, K = keep.
[g] Animal group applied to. Country parameters have been estimated from given in parentheses.
[i] Management information system.

6 Software for solving MDP models

The value iteration algorithm for ordinary MDPs is relatively easy to implement and most papers have implemented the algorithm using various programming languages. The policy iteration is harder to implement since we have to invert a matrix when solving the set of linear equations. That is probably the reason that most studies reported in literature have used the more straightforward value iteration algorithm. In a few cases software packages in MATLAB[1] have been used to perform policy iteration (Heikkila et al, 2008, 2012). Linear programming can also be used to find optimal policies but have only been used in two papers (Cabrera, 2010; Yates and Rehman, 1998).

When considering hierarchical MDPs implementation becomes harder due to the nested structure of the processes. Fortunately a general software system MLHMP for construction, editing and optimization of Markov decision processes ranging from finite time ordinary MDPs to hierarchical MDPs has been developed by Kristensen (2003). MLHMP is implemented in Java[2] with the possibility of building models as plug-ins. Moreover, it can handle all the criteria mentioned in this paper. MLHMP has been used to solve almost all hierarchical MDPs in the literature. Recently, a package "Markov decision processes (MDPs) in R" (Nielsen, 2011) has been developed for model building in R[3]. It is based on a C++ implementation for fast execution of policy and value iteration and can be used to solve both ordinary and hierarchical MDPs under all criteria.

7 Conclusions and directions for further research

In this chapter MDPs have been considered to model livestock systems. Livestock farming problems are often sequential in nature and hence MDPs are suitable as a modeling tool.

A total of approximately 80 papers using a MDP for modeling the livestock system have been reviewed with the first paper dating back to 1966 and the last paper in 2012. Only decision models are included in the survey, i.e., simple Markov chain models are not mentioned even though they are, of course, closely related to MDPs. Most papers have been considered within dairy and some within pig production; however, MDPs have also been applied to other areas.

The papers may be divided into two categories, namely, papers using MDPs as a tool for evaluating different herd effects, e.g., different reproductive programs (Kalantari and Cabrera, 2012) and papers formulating MDP models which may be embedded into a management decision support system (*DSS*), e.g., a model for slaughter pig marketing (Kristensen et al, 2012).

The first category is mainly used by researchers as an evaluation tool and giving advise to the industry. Several of the most advanced recent models are in this category. Thus, the models by Bar et al (2008a,b) and Cha et al (2011) use the models to estimate the costs of clinical mastitis in dairy cows and evaluate the treatment and prevention options, and Demeter et al (2011) use their

[1] MathWorks Inc. `http://www.matlab.com`.
[2] Oracle `http://www.java.com/`.
[3] R Development Core Team `http://www.R-project.org/`

model to estimate the long-term consequences of different breeding strategies in dairy cows. It is expected that many models developed in the future will belong to this category.

The aim of models in the second category is that they ultimately should be used within the DSS on farm. However, the actual use of such models on farm has been limited. Reasons for this may be that MDPs require access to good data for estimating the many parameters needed in the model. Moreover the estimation process may be cumbersome and error-prone. As a result there have been a growing focus on using on-farm biosensors for retrieving data and algorithms for data filtration and parameter estimation based on Bayesian updating as in (Nielsen et al, 2010) for a dairy cow replacement model. An example from pig production is the work by Bono et al (2012) where important litter size parameters to be used in a sow replacement models are automatically and dynamically estimated from herd registrations and fed into the replacement model. Furthermore, the states of the individual sows are automatically identified so that the optimal decision can be returned by the optimization model. Providing direct links from data is crucial if MDP models should be applied within farms since the parameter settings may be quite different among farms.

Another issue is violated herd constraints. MDP models are often applied at animal level and given replacement it is assumed that a new animal is available. As a result MDP models have to be coordinated with other information streams and other models used in the farm DSS. This calls for further research.

Due to the large number of state variables there is a trend in using hierarchical MDPs since here state variables such as lactation number and lactation stage are implicitly given by the model structure. Hence, the same problem formulated as a hierarchical model will typically have fewer state variables than if it had been formulated as an ordinary MDP. Moreover, finding the optimal policy using policy iteration is often faster.

Finally, the number of state variables may be so large that models may face the curse of dimensionality. This calls for research in models which finds an approximate good policy using techniques such as approximate dynamic programming (Powell, 2011).

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