Inventory control in a lost-sales setting with information about supply lead times*

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Abstract: Supply chain collaboration using advancements in information technology is on the rise and this includes sharing of information between suppliers and buyers. In this paper we study the value of information about the development of supply lead times from a buyer's perspective. We consider a periodically reviewed single-item inventory system in a lost sales setting where at most one order can be outstanding at a time. We compare the performance of an inventory model assuming informed lead times to a model assuming uninformed independent and identically distributed lead times. We employ the dynamic programming approach to find the best state-dependent ordering policy to minimize the expected average total cost per time unit. Our numerical results show that acquiring information about the development of supply lead times is of value. In general the best policy suggested by the model assuming informed lead times causes lower average cost than the model assuming uninformed lead times.

Keywords: Lost sales, Stochastic lead times, Informed lead times, Dynamic programming, Markov decision process

1 Introduction

The crucial part of inventory management is to make replenishment decisions in the face of uncertainties at upstream and downstream stages of the supply chain. Demand uncertainty represents uncertainty in the downstream supply chain, whereas lead-time uncertainty captures the uncertainty of the supply system, i.e. the upstream part of the supply chain. As discussed in an empirical examination by Wagner and Bode (2008), such variations can be equated to supply chain risks which lower its performance. One way to mitigate supply chain risk is by improving confidence between decision-makers through collaboration and better information sharing (Christopher and Lee, 2004). An empirical study by Li, Yang, Sun, and Sohal (2009) suggests

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that effective use of information technology has a positive effect on supply chain integration and hence on its performance. Advancements in information technology and its role in supply chain management provides managers with an opportunity to relatively easily obtain dynamic information about demand and supply variations. Similar to demand variations, major variations in supply lead times may have identifiable sources, such as equipment breakdown, workload conditions etc. These sources of variation reflect the condition of the supply system, and replenishment lead times evolve as the system evolves over time. Inventory models which consider the evolution of demand and use advance information in decision making are widely studied, for example Hariharan and Zipkin (1995); Karlin and Fabens (1959); Song and Zipkin (1993) and Gallego and Özer (2001). However, more sparsely studied are the models with shared information about upstream supply conditions. The hypothesis test by Cannon and Homburg (2001) shows that an increasing number of suppliers share information to reduce acquisition and operation costs for the customer, thus emphasizing a need for further study of such systems.

Since the mid-1990s some results for inventory control models with information about supply conditions have been available, such as Song and Zipkin (1996). In these models, stockouts are backordered. However, stockouts may also result in lost sales. The lost-sales case has considerable practical significance. The study by Corsten and Gruen (2004) shows that in almost half of the cases unmet demands result in lost sales not backorders. Lost sales also appear to be a common mechanism for handling shortages in some spare parts industries.

The purpose of this paper is to investigate the performance of the value of advanced lead-time information under lost sales. We study a single-item inventory control model with lost sales and ubiquitous demand uncertainty, controlled by a periodic review inventory replenishment policy with at most one order outstanding at any given time. The inventory is supplied by a system with evolving replenishment lead times which are informed in advance and with certainty at every ordering decision epoch.

The assumption of no more than one outstanding replenishment order at a time can be justified from a practical standpoint. The likelihood of placing a new order while awaiting the arrival of an existing order can be negligible due to, for example, terms in the supplier-buyer contract or due to the buyer's internal ordering policy. Moreover, allowing for simultaneously outstanding orders makes the model numerically intractable in general (Zipkin, 2008a,b).

We assume that the supply system is exogenous, i.e. its development is independent of our demands and replenishment orders (Zipkin, 2000, Section 7.4). Hence, replenishment order lead time is independent of the size of our order and only depends on the conditions of the supply system. However, the system is transparent in the sense that it is assumed to be possible to obtain information about its current state and about the development of its supply lead times. Hence, changes in lead times are characterized by a Markov chain. Thus, with a reliable estimate of current lead time, information of the development of future lead times is available through the Markovian dependence. The performance of the model with information about the development of supply lead times is compared to the model with independent and identically distributed (i.i.d.) lead times. The latter model represents the case where the supplier is not sharing information about the development of the supply lead times.

We consider an infinite time horizon inventory problem and formulate it as a dynamic program with finite and discrete state space. We assume that ordering, inventory holding and lost sales penalty costs are linear and the objective is to minimize expected average total cost per time unit over an infinite time horizon. Through numerical experiments, we observe the significance of information about the development of lead times. By changing the evolutionary pattern, we are able to study its qualitative effect on the inventory performance assuming either informed lead times or i.i.d. lead times. Such comparisons enable us to estimate the value of information available about the development of lead times compared to knowing only their long-term distribution.

The rest of the paper is organized as follows. In Section 2 we give a brief survey of the literature related to inventory systems with lost sales and information about supply conditions. Analytical models used for our numerical study are introduced in Section 3 and computational experiments are specified and reported in Section 4. Finally, Section 5 summarizes the results and concludes the paper.

2 Literature review

Davis (1993) contains a general discussion about the role of uncertainty in supply chain management from a business perspective by using Hewlett-Packard as a case study. Not only upstream and downstream uncertainties complicate the management of inventory but the way to treat stockouts also further complicates the problem. Inventory models with backlogging of unfulfilled demand are reasonably well understood today and detailed discussion about them can be found in Zipkin (2000) and Axsäter (2006). However, inventory control models in which stockouts are treated as lost sales have been studied more sparsely. For further insight into the differences between models with backorders and lost sales, we can refer to Montgomery, Bazaraa, and Keswani (1973) who study a model in which, during a stock out, a fraction of the demand is backordered and the rest is lost. The basic problem of an inventory model with lost sales (constant lead time) was formulated more than 50 years ago by Karlin and Fabens (1959). Yet, today it proves to be a difficult problem to work with because of the rapidly growing state space with longer lead times, which is discussed in Zipkin (2000, Section 9.6.5) and described by the *curse of dimensionality*. Hence, few extensions to more complex systems are available.

Hadley and Whitin (1963, Section 4-11) discuss the difficulty in analyzing a continuously reviewed reorder point policy under lost sales. For results regarding periodically reviewed reorder point policies under lost sales see Johansen and Hill (2000). Nahmias (1979) provides approximations for the periodically reviewed inventory model with non-linear ordering cost and variable lead times, while Cohen, Kleindorfer, and Lee (1988) present an approximation for the periodically reviewed (s,S) inventory model with two priority demand classes.

Downs, Metters, and Semple (2001) study a multi-item periodically reviewed model with deterministic delivery time lags and a finite horizon. Recently, Zipkin (2008b) has presented an elegant state-space reduction technique for inventory models with lost sales and extended this approach to important variations of the model: limited capacity, correlated demands, stochastic lead times, and multiple demand classes. Equipped with this state-space reduction technique, Zipkin (2008a) is able to obtain optimal replenishment policies for longer lead times than reported before. His paper also tests plausible heuristics for a limited range of systems.

Due to the relatively easy-to-handle structure of inventory models with backordering they have usually been considered in past studies assuming more complex settings, such as information about supply conditions. This is the case, for example, in Song and Zipkin (1996), where exogenous supply is modeled as a Markov chain and as the information about supply evolves so do lead times. They analyze the effect of evolving lead times on policy decisions. Özekici and Parlar (1999) assume that the order is satisfied immediately if the supplier is available and, in the other extreme, if the supplier is unavailable, the order is never fulfilled. Availability of the supplier depends on the environment, which is modeled as a Markov chain. Gallego and Hu (2004) analyze inventory problems with Markov-modulated supply with limited capacity as well as separately Markov-modulated demand processes. Arifoğlu and Özekici (2010) extend this model and its results by considering the case where available information is imperfect due to a partially observable environment.

The paper by Ben-Daya and Hariga (2004) discusses the single-vendor-single-buyer production inventory model which minimizes the consolidated expected total cost per time unit for vendor and buyer. They assume that the lead time of an order depends on its size. A comment on this paper by Glock (2009) takes the model a step further by demonstrating benefits in considering different reorder points for each batch shipment and unequal-sized batch shipments. A note on the same paper by Hsiao (2008) proposes a variation of the original model by assuming two different reorder points and service levels.

For the reasons mentioned above, inventory models under lost-sales settings and with information about supply conditions have received limited attention. Arreola-Risa and DeCroix (1998) explore the model with supply durations occurring randomly and lasting for a random duration. Their model assumes that the demand is stochastic and delivery lead times are equal to zero. In case of stockouts, a fraction of the demand is backordered and the remaining fraction is lost. In the paper by Mohebbi (2003), he investigates the issue of random supply interruptions (available/unavailable) in a continuous review inventory system where demand as well as non-zero lead times are stochastic and where stockouts are lost sales. He assumes that the duration of supplier availability and unavailability is independent. It is also assumed that the maximum number of outstanding replenishment orders is limited to one at any time. Variations of this model are presented by Mohebbi (2004) and Mohebbi and Hao (2006). In all these models, the assumption is that supply interruptions and the duration are random and independent. Hence, although information about availability of the supplier is provided at the current moment, no information about future supply conditions is available. However, Li, Xu, and Hayya (2004) present a periodic review inventory model in a lost-sales setting, which allows for the age of the availability of the supplier to affect availability in the next time period. Thus, in this model some information about the future condition of the supplier is provided, given that the supplier is available at the current moment. The model assumes that the lead time is negligible and that a replenishment order can be placed only if the supplier is available. Thus, there appears to be a basis for exploring inventory control under the lost-sales setting, where the information available about supply conditions not only informs about the current lead time, but also provides some information about its future development.

3 Model formulation

We consider a periodically reviewed inventory model under a lost-sales setting and assume that there can never be more than one outstanding order at any time. We describe the inventory model using the following notation:

- n Index denoting the decision epoch under consideration
- x_n Inventory level after order due in decision epoch n is delivered
- o_n Quantity outstanding in decision epoch n
- u_n Order delivered in decision epoch n
- M Maximum possible inventory (on hand + on order)
- L_n Lead time at decision epoch n
- Z Maximum possible lead time of an order
- D_t Cumulative demand over t time units. We use $D = D_1$ as short-hand notation
- q_n Replenishment order placed at decision epoch n
- r_n Time elapsed for the outstanding order at decision epoch n
- c Unit cost of procurement
- h Unit inventory holding cost per time unit
- p Unit penalty cost for lost sales

As in Song and Zipkin (1996), we may suppress the index n and write x for x_n and x_+ for x_{n+1}, i.e. the subscript $+$ denote the next decision epoch. Moreover, we define $x^+ = \max(0,x)$ and the indicator function $\mathbf{1}_{\{v \in V\}}$ to be equal to one if $v \in V$ and zero otherwise.

To find the optimal inventory policy, we model the inventory system as a discrete infinite time-horizon semi-Markov decision process (sMDP). At decision epoch n the system occupies a *state* s. Given the decision-maker observes state s at decision epoch n, he can choose a *decision* q from the set of allowable decisions generating *expected cost* c_s^q until the next decision epoch. The length of the decision epoch is stochastic and dependent on q and s. Moreover, let p_{s,s_+}^q denote the *transition probability* of obtaining state s_+ at the next decision epoch. A *policy* specifies which decision q to use at all states s for all decision epochs and provides the decision maker with a plan of which decision to take given decision epoch and state. The optimal policy is found under the *average expected cost criterion*. Moreover, since the state and decision space are finite and the sMDP is stationary and unichain, the optimal policy can be found using *policy iteration* (Puterman, 1994).

3.1 Model with informed supply lead times

In this model, reliable information about the lead time of an order placed in the current period is available. At each decision epoch, the sequence of events is as follows:

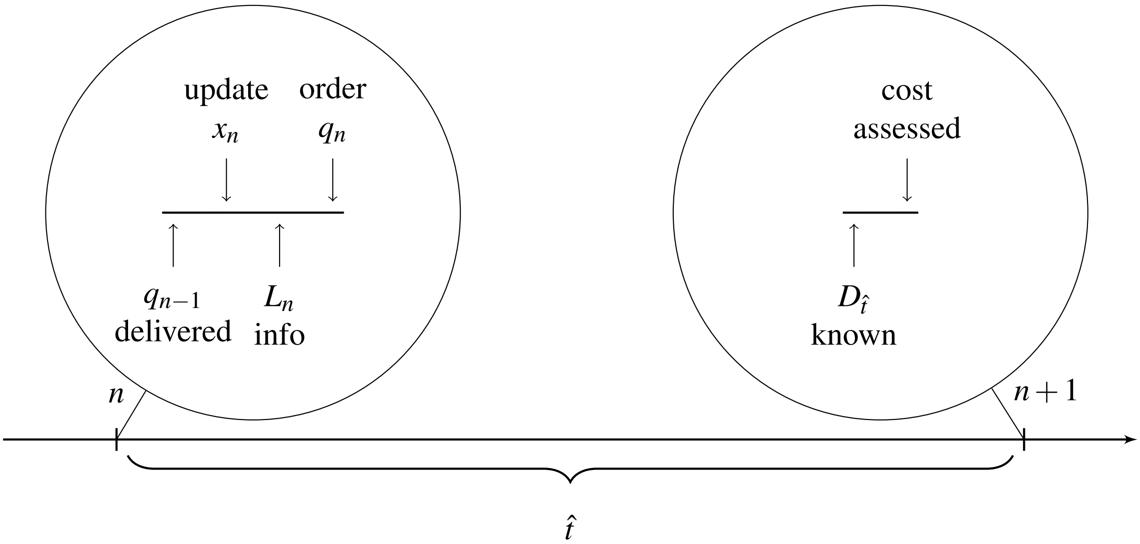


Figure 1: The events at decision epoch n (informed lead times).

1. If an order is due, then it is delivered, and the state of the inventory level is reviewed. The supplier provides reliable information about the lead time of an order placed in the current period.
2. A decision about placing a new replenishment order is made.
3. Demand occurs and the procurement, holding and lost-sales penalty costs are assessed.

When making decision q, the time until the next decision epoch is

$$\hat{t} = \hat{t}(q,L) = \begin{cases} L & \text{if } q > 0, \\ 1 & \text{if } q = 0. \end{cases} \tag{1}$$

For the decision epoch at time t, state (x,L) and order q, the next decision epoch occurs at time $t+\hat{t}$. Thus, we have decision epochs of various length. Moreover, we assume that the length of a review period is chosen such that the lead time is at least one time unit. Decision epoch n is illustrated in Figure 1.

The inventory dynamics are given by

$$x_+ = (x - D_{\hat{t}})^+ + q,$$

To find the optimal inventory policy, we model the inventory system as a discrete infinite time-horizon semi-Markov decision process. A state $s = (x,L)$ is defined using state variables x and L where $x \in \{0,\dots,M\}$ and $L \in \{1,\dots,Z\}$. The set of possible order quantities (decisions) q is $\{0,\dots,M-x\}$. The time between each decision epoch is given in (1) Note that the restriction on the maximum possible inventory (M) and lead-time (Z) allows us to have a finite state space as well as a finite decision set. High values of M and Z make the number of states high which may pose a problem due to the curse of dimensionality; however, limited maximum inventory can be valid in real-life situations, for example due to budget constraints or constraints on physical storage capacity. Very long lead times may occur, e.g. due to a strike or supplier breakdowns, but are rare and not handled by the model.

The transition probabilities can be calculated as

$$p^q_{s,s_+} = \Pr(x_+, L_+ \mid x, L, q, \hat{t}) = \Pr(x_+ \mid x, q, \hat{t}) \cdot \Pr(L_+ \mid L, \hat{t}), \tag{2}$$

where

$$\begin{aligned}
\Pr(x_+ \mid x, q, \hat{t}) &= \Pr((x - D_{\hat{t}})^+ + q = x_+) \\
&= \begin{cases} \Pr((x - D_{\hat{t}})^+ = 0), & \text{if } x_+ = q, \\ \Pr((x - D_{\hat{t}}) + q = x_+), & \text{if } x_+ > q, \end{cases} \\
&= \begin{cases} \Pr(D_{\hat{t}} \geq x), & \text{if } x_+ = q, \\ \Pr(D_{\hat{t}} = x + q - x_+), & \text{if } x_+ > q, \end{cases}
\end{aligned}$$

corresponding to a demand higher and below the current inventory level. Note that the current lead time is deterministic; however, the lead time at the next decision epoch is a random variable dependent on the current lead time. The probability $\Pr(L_+ \mid L, \hat{t})$ can be calculated based on a function of a random variable (see Section 4).

The cost incurred until the next decision epoch is given by

$$c^q_s = cq + H(x, \hat{t}), \tag{3}$$

i.e. the procurement cost and $H(x, \hat{t})$ which denotes the expected holding and lost-sales cost over $\hat{t}$ time periods given the present inventory level x. H can be defined recursively as

$$\begin{aligned}
H(x,t) = \sum_{d=0}^{x} \Pr(D = d)(h(x-d) + H(x-d, t-1)) \\
+ \sum_{d=x+1}^{\infty} \Pr(D = d)(p(d-x) + H(0, t-1))
\end{aligned}$$

with $H(x,0) = 0$. A discount factor equal to 1 is assumed. A fixed ordering cost may be added to (3) which in general will result in an optimal policy where small orders are avoided; however, for the simplicity of the model we choose not to include it in the model. Moreover, a fixed ordering cost is usually associated with the administration of placing a replenishment order. With advancements in automation of inventory management systems the cost of placing an order is becoming minimal. Hence, from a practical point it is valid to assume the fixed ordering cost to be zero.

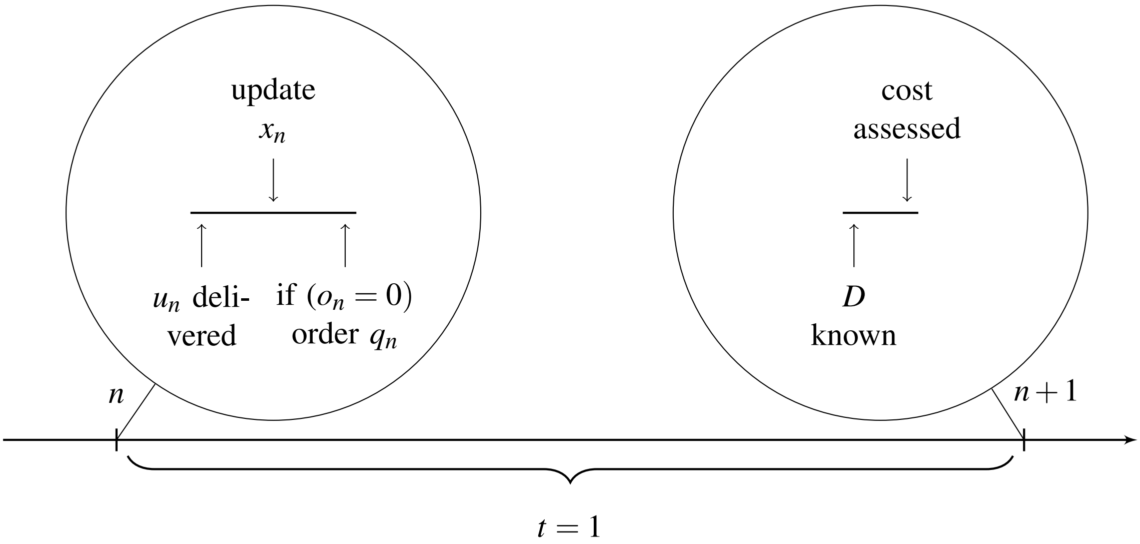


Figure 2: The events in the decision epoch n (stochastic lead times).

3.2 Model assuming stochastic supply lead times

In this alternative model, the lead times are assumed to be i.i.d. and no certain information is available about the lead time of an outstanding order. In each period, the sequence of events is as follows:

1. If an order is due in the period, then it is delivered and the state of the inventory level is reviewed.
2. If an order is not outstanding already, then a decision about placing a new replenishment order is taken.
3. Demand occurs and procurement, holding and lost-sales penalty costs are assessed.

Unlike in the previous model, we need to review the inventory level state in every period in this model, i.e. the length of each decision epoch is one time unit. If in the current period a previously outstanding order is delivered, then a new ordering decision can be made. Otherwise, the only possible decision is not to place an order. The decision epoch is illustrated in Figure 2. The inventory dynamics are given by

$$x_+ = (x - D)^+ + u_+.$$

To find the optimal inventory policy, we model the inventory system as a discrete, infinite time-horizon Markov decision process. A state $s = (x, o, r)$ is defined using state variables x, o and r, where $x \in \{0, \ldots, M\}$, $o \in \{0, \ldots, M - x\}$ and $r \in \{0, \ldots, Z - 1\}$. Note that not all combinations of the state variables are possible since o and r are interrelated. If there is no outstanding order or an order is received then $o = r = 0$. Otherwise, if there is an order outstanding, then $o > 0$ and $r > 0$. If $o = 0$, the set of possible order quantities (decisions) $q \in \{0,\dots,M-x\}$. Moreover, since only a single order can be outstanding, no order can be issued ($q = 0$) if $o > 0$.

Let $p^q_{ss_+} = \Pr(x_+, o_+, r_+ \mid x, o, r)$ denote the transition probability. If $o = r = 0$ and $q = 0$ then

$$p^q_{ss_+} = \begin{cases} \Pr(D \geq x), & \text{if } x_+ = 0, o_+ = 0, r_+ = 0 \\ \Pr(D = x - x_+), & \text{if } x_+ \in \{1,\dots,x\}, o_+ = 0, r_+ = 0 \\ 0, & \text{otherwise,} \end{cases}$$

corresponding to a demand higher and lower than the current inventory. If $o = r = 0$ and $q > 0$, then

$$p^q_{ss_+} = \begin{cases} \Pr(D \geq x)\Pr(L = 1), & \text{if } x_+ = q, o_+ = 0, r_+ = 0 \\ \Pr(D = x - x_+ + q)\Pr(L = 1), & \text{if } x_+ \in \{q+1,\dots,q+x\}, o_+ = 0, r_+ = 0 \\ \Pr(D \geq x)\Pr(L > 1), & \text{if } x_+ = 0, o_+ = q, r_+ = 1 \\ \Pr(D = x - x_+)\Pr(L > 1), & \text{if } x_+ \in \{1,\dots,x\}, o_+ = q, r_+ = 1 \\ 0, & \text{otherwise,} \end{cases}$$

corresponding to a demand higher and lower than the current inventory and arrival of the order at next decision epoch and a demand higher and lower than the current inventory and arrival of the order after next decision epoch, respectively. When $o > 0$, $r > 0$, then $q = 0$ and we have that

$$p^q_{ss_+} = \begin{cases} Pr(D > x)Pr(L = r+1 \mid L > r), & \text{if } x_+ = o, o_+ = 0, r_+ = 0 \\ Pr(D = x - x_+ + o)Pr(L = r+1 \mid L > r), & \text{if } x_+ \in \{o+1,\dots,o+x\}, o_+ = 0, r_+ = 0 \\ \Pr(D \geq x)Pr(L > r+1 \mid L > r), & \text{if } x_+ = 0, o_+ = o, r_+ = r+1 \\ Pr(D = x - x_+)Pr(L > r+1 \mid L > r), & \text{if } x_+ \in \{1,\dots,x\}, o_+ = o, r_+ = r+1 \\ 0, & \text{otherwise,} \end{cases}$$

corresponding to a demand higher and lower than the current inventory and arrival of the order at next decision epoch and a demand higher and lower than the current inventory and arrival of the order after next decision epoch, respectively. The probabilities $\Pr(L = l)$ are derived from the steady-state probabilities of the corresponding model with informed lead times (see Section 4). Hence, the resulting lead-time distributions used are the same in both models.

The expected cost until the next decision epoch is given by

$$c^q_s = cq + h\mathbb{E}\left((x-D)^+\right) + p\mathbb{E}\left((D-x)^+\right),$$

i.e. the procurement cost and expected holding and lost-sales cost of one time unit. The mean of $(x-D)^+$ can be calculated as

$$\mathbb{E}\left((x-D)^+\right) = \sum_{d=0}^{x} (x-d)\Pr(D = d),$$

since D is discrete and non-negative. Moreover, $\mathbb{E}\left((D-x)^+\right) = \mathbb{E}\left((x-D)^+\right) - x + \mathbb{E}(D)$.

4 Numerical results

The computational experiments were conducted using R (R Development Core Team, 2010) together with the R package "MDP" (Nielsen, 2009), which was used to generate each MDP and solve the model using policy iteration under the average cost criterion. The experiments provide insights into which parameters of the model affect the reduction

$$\Delta = 100(1 - g_I/g_U), \tag{4}$$

i.e. the percentage reduction of the average cost per time unit under informed lead times (g_I) compared to the average cost per time unit under uninformed lead times (g_U).

In our experiments we assume that $p > c$. Think of p as the unit cost of lost revenue or goodwill or the cost of a special emergency delivery. Given that possibility, if $p < c$, then it is obviously preferred to always use an emergency source instead of a regular source. Hence, it would be appropriate to assume $p > c$. In fact, in most of our experiments (instances 1-30) we also assume that $p > c + h$. Thus, it would not be cheaper to lose demands occurring in the next period than to carry inventory forward. This kind of cost structure can typically be found in the retail industry where lost sales are commonly observed. A cost structure where $p \leq c+h$, i.e. the cost of carrying a specific inventory are comparably high, were also tested which apply to other industries (instance 31-36).

Preliminary tests showed that a fixed ordering cost added to (3) has little effect on the cost ratio (4) between the two models and the optimal policy. As a result we only consider test instances without a fixed ordering cost in the experiments.

We assume that the demand is Poisson distributed ($D \sim \text{Po}(\lambda)$). Moreover, in the case of informed lead times, we assume that the transition over one time unit is given by:

$$L_{t+1} = \begin{cases} 1, & \text{if round}(L_t + B_t) \leq 1 \\ \text{round}(L_t + B_t), & \text{if round}(L_t + B_t) = \{2, \dots, Z-1\}, \\ Z, & \text{if round}(L_t + B_t) \geq Z \end{cases} \tag{5}$$

where $B_t \sim N(\mu, \sigma^2)$ and hence the transition probabilities become:

$$\Pr(L_{t+1} \mid L_t) = \begin{cases} \Pr(B_t \leq 1.5 - L_t), & \text{if } L_{t+1} = 1 \\ \Pr(B_t \in [L_{t+1} - L_t - 0.5, L_{t+1} - L_t + 0.5], & \text{if } L_{t+1} \in \{2, \dots, Z-1\} \\ \Pr(B_t \geq Z - L_t - 0.5), & \text{if } L_{t+1} = Z \end{cases} \tag{6}$$

Let P denote the transition matrix of (6) then $\Pr(L_+ = i \mid L = j, \hat{t})$ in (2) can be calculated as the (i, j)'th element in $P^{\hat{t}}$ (Puterman, 1994).

The choice of (5) and (6) is based on the observation that they provide a discrete distribution on a finite set of lead times $L \in \{1, \dots, Z\}$. Moreover, by modifying the mean and variance of B, we can change the shape of the steady-state distribution (used in the model with stochastic lead times) of L to be close to uniform or having a high probability mass at the endpoints. This can be

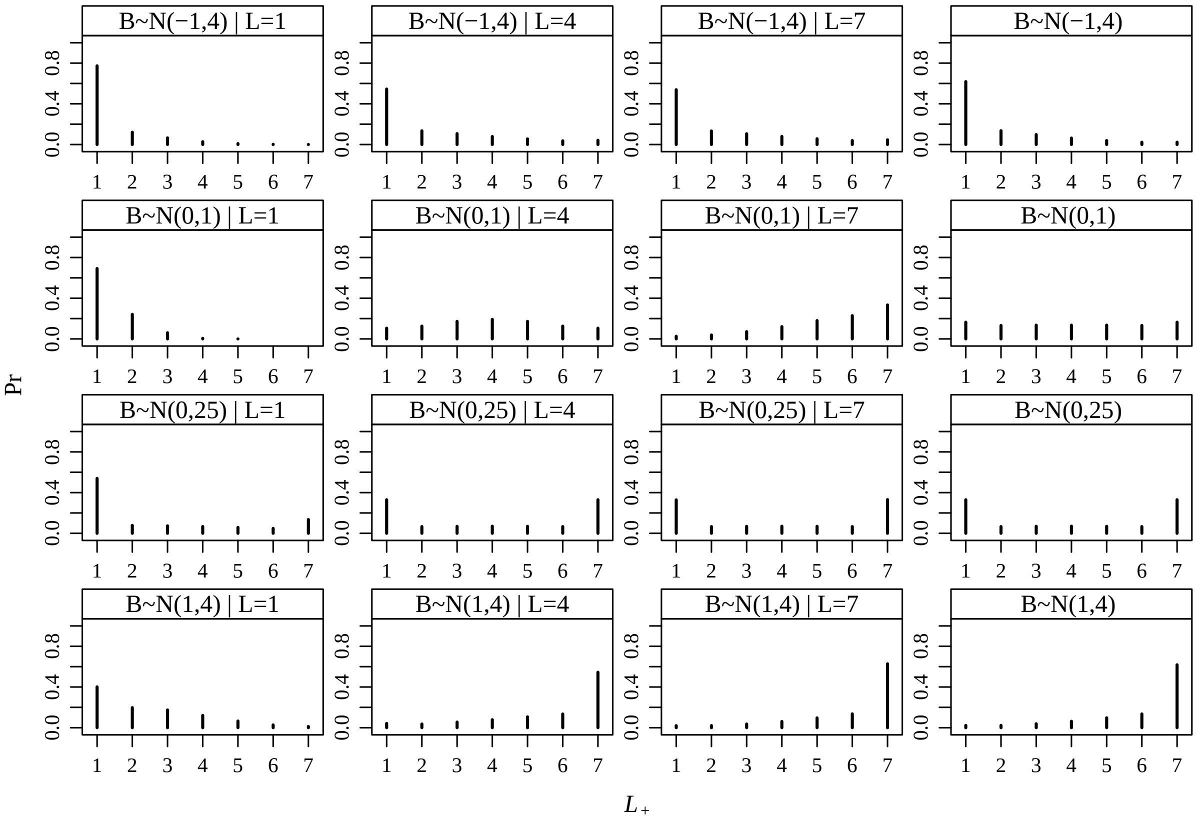


Figure 3: The transition probabilities $\Pr(L_+ \mid L, \hat{t} = L)$ given $L = 1,4,7$ in the model with informed lead times and the corresponding steady state probabilities (last column) used in the model with stochastic lead times (for $Z = 7$). Each row represents a different combination of mean and variance of B.

seen in Figure 3, where the probability mass function of $\Pr(L_+ \mid L, \hat{t} = L)$ for $L = 1,4,7$ and $Z = 7$ and the steady-state probabilities are illustrated. Note that zero mean and high variance results in a high probability mass at 1 and Z. Conversely, a small variance results in a near uniform steady-state distribution. Letting the mean be different from zero gives a bias towards either 1 or 7.

The experiments are shown in Table 1. First, we examine the effect of increasing the penalty cost p using instances 1-6. This is illustrated in Figure 4a. Increasing p makes the average cost per time unit g increase for both models. Moreover, the benefit of informed lead times (Δ) increases with p.

In test instances 7-15, the effect of the demand rate (λ) is considered. The results are illustrated in Figure 4b. The reduction Δ peaks around $\lambda = 4$. If the demand rate is low, then lost sales are rare under the optimal policy and we cannot utilise our information about the lead times very much to avoid penalty cost. If the demand rate is very high, then due to the maximum

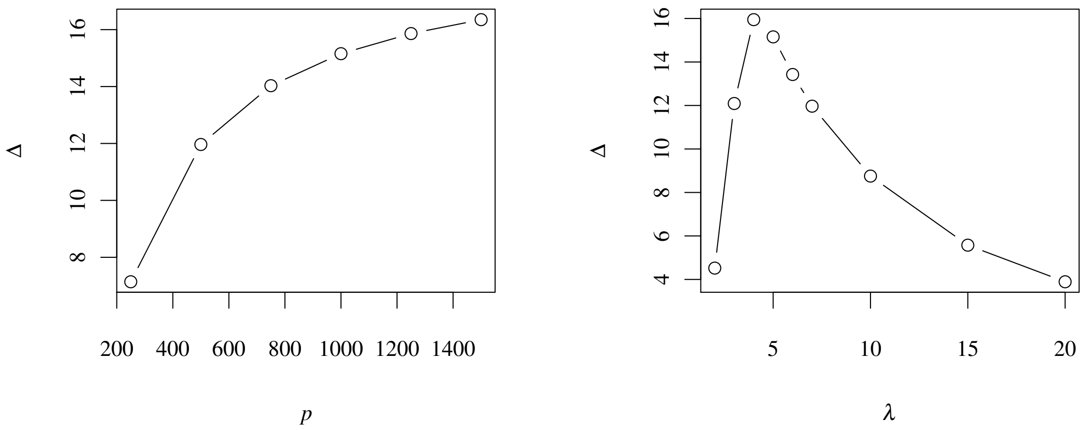
Table 1: Results for different instances of the model.

#	M	Z	μ	σ	λ	c	p	h	g_I[a]	g_U[b]	Δ[c]
1	20	7	0	1	5	100	250	10	842	907	7.1
2	20	7	0	1	5	100	500	10	1353	1537	12.0
3	20	7	0	1	5	100	750	10	1863	2167	14.0
4	20	7	0	1	5	100	1000	10	2372	2796	15.2
5	20	7	0	1	5	100	1250	10	2881	3424	15.9
6	20	7	0	1	5	100	1500	10	3390	4053	16.4
7	20	7	0	1	2	100	1000	10	440	461	4.5
8	20	7	0	1	3	100	1000	10	985	1121	12.1
9	20	7	0	1	4	100	1000	10	1649	1962	15.9
10	20	7	0	1	5	100	1000	10	2372	2796	15.2
11	20	7	0	1	6	100	1000	10	3141	3627	13.4
12	20	7	0	1	7	100	1000	10	3947	4484	12.0
13	20	7	0	1	10	100	1000	10	6540	7168	8.8
14	20	7	0	1	15	100	1000	10	11209	11870	5.6
15	20	7	0	1	20	100	1000	10	15964	16611	3.9
16	20	7	0	1	5	100	1000	10	2372	2796	15.2
17	20	7	0	5	5	100	1000	10	2060	2802	26.5
18	20	7	0	10	5	100	1000	10	1676	2805	40.2
19	20	7	0	15	5	100	1000	10	1524	2805	45.7
20	20	7	0	20	5	100	1000	10	1446	2805	48.4
21	20	7	0	25	5	100	1000	10	1400	2806	50.1
22	20	7	0	50	5	100	1000	10	1306	2806	53.5
23	20	7	1	5	5	100	1000	10	2479	3000	17.4
24	20	7	-1	5	5	100	1000	10	1635	2545	35.7
25	20	5	0	1	5	100	1000	10	1964	2292	14.3
26	20	7	0	1	5	100	1000	10	2372	2796	15.2
27	20	10	0	1	5	100	1000	10	2764	3182	13.1
28	20	12	0	1	5	100	1000	10	2952	3360	12.2
29	20	15	0	1	5	100	1000	10	3169	3567	11.2
30	20	20	0	1	5	100	1000	10	3429	3814	10.1
31	20	7	0	1	5	100	150	50	683	699	2.3
32	20	7	0	1	5	100	150	60	688	703	2.2
33	20	7	0	1	5	100	150	70	692	706	2.0
34	20	7	0	1	5	100	150	80	695	709	2.0
35	20	7	0	1	5	100	150	90	697	711	1.9
36	20	7	0	1	5	100	150	100	700	712	1.8

[a] Average cost per time unit (informed lead times).
[b] Average cost per time unit (uninformed lead times).
[c] Cost reduction in percent ($100(1 - g_I/g_U)$).

inventory level M, we will almost always have lost sales and the information about lead times will not solve this problem. Hence, a demand rate in-between gives the best benefit of informed lead times.

For test instances 16-24, we look at different lead time distributions. The different distributions can be studied in Figure 3. In instances 16-22, we increase the variance of B used in (6). Higher variance results in more fluctuations in lead time at two subsequent epochs and higher probability mass at the end points (1 and Z) of the steady-state distribution (see Figure 3). Different distributions have a large impact on Δ. A cost reduction of approximately 50% is obtained if the distribution has a high probability mass at the end points. The model assuming informed lead times can utilise the transition probabilities in (6) better and hence produce results adjusted to varying development patterns of L. This can be seen in Figure 5, where Figure 5a shows the optimal policy for $\sigma = 1$ (near uniform steady-state distribution) and Figure 5b the optimal policy



(a) Penalty cost p.

(b) Demand rate λ.

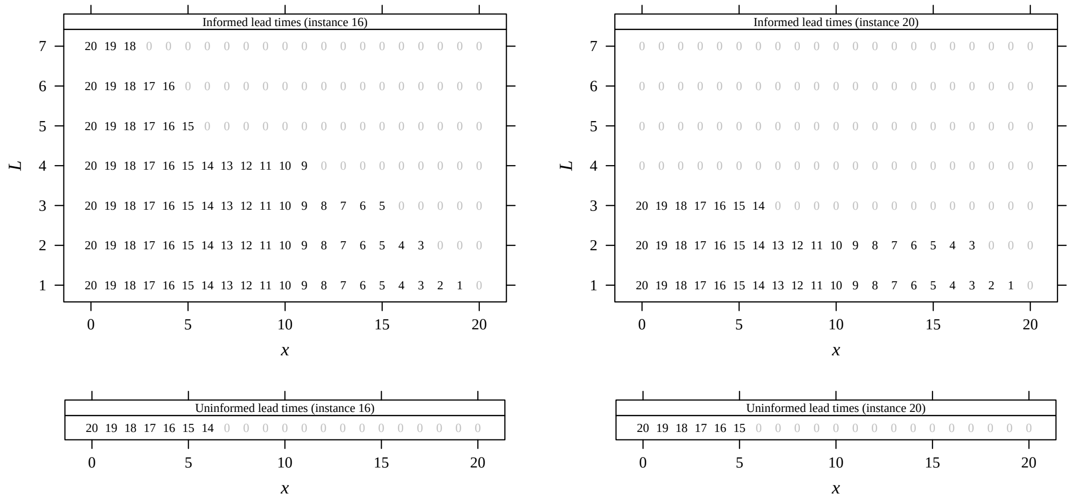
Figure 4: Development of Δ for different parameters.

for $\sigma = 20$ (high probability mass at the end points). Note that in the case with high probability mass at the end points the optimal policy only orders for low lead times, whereas in the other case orders are placed for high lead times, too. With uninformed lead times, the optimal policy is almost identical in both cases.

Instances 23 and 24 provide results when we have a trend towards a lead time of either one or Z. The average cost reduction is much higher when we have a bias towards a low lead time. This is due to the fact that if we often have a low lead time, the optimal policy can benefit more from this in the case of informed lead times.

The range of the lead-time distribution when the steady-state distribution is close to uniform is considered in instances 25-30. Increasing Z implies decreasing benefit of informed lead times (Δ); however, the effect is not considerable compared to the structure of the lead time density (instances 16-24). By increasing the range, the average inventory cost will increase as well.

Finally, we have a look at a cost structure where $p \leq c+h$, i.e. the cost of carrying a specific inventory is comparably high (see instance 31-36 in Table 1). In this case the benefit of informed lead times is lower compared to the case where $p > c+h$. When $p < c+h$, it would be cheaper to lose sales than to carry inventory into the next period. In that case, the informed lead time model only uses the information about the current state L and does not really use the information about the conditional density (6) as the model would only suggest to order enough to cover the demand in one period. Thus the information available about lead times is not used to its potential and hence the improvement achieved is smaller compared to the case where $p > c+h$.



(a) $\sigma = 1$.

(b) $\sigma = 20$.

Figure 5: Optimal policy for two different values of σ. Each number represents the optimal order amount given the state. In the uninformed case, only states with $r = o = 0$ are shown.

5 Conclusions and future research

Unlike the conventional assumption in inventory models that supply lead times are independent and/or identically distributed, we have considered a model in which lead times are dependent and where information about the development of lead times is available. These lead times are modeled as a Markov chain and the information about their development is represented by transition probabilities. We have studied a periodically reviewed lost-sales inventory model with stochastic demand and compared the performance of the model assuming dependent and informed lead times with the model assuming i.i.d. lead times.

Our numerical results show that information regarding the development of lead times may have a significant effect on inventory replenishment decisions. Using information about lead times can considerably improve the performance of the inventory control model compared to considering lead times only to be i.i.d.

Informed lead times seem to have the most significance when the demand rate is neither too low nor too high (cf. Figure 4) and when the steady-state distribution of the lead time has a high probability mass at a low lead time. Also, using information about lead times seems to be most beneficial when lost sales are relatively costly.

Inspired by the integrated single-vendor-single-buyer production inventory model studied in Glock (2012), an interesting extension study could be to assume that the lead time of an order depends on its size and the lead time of an order can be reduced for an extra cost. Moreover, as discussed in Glock and Ries (2012) and Guiffrida and Jaber (2008) there are ways to reduce supply chain variances by investing in extra resources and a big impact can be achieved by doing so. This could be a potential extension of the paper. Also, when considering longer lead times, allowing multiple outstanding orders will significantly increase the complications in solving this problem. Hence, a small improvement step could be to consider at most two orders outstanding instead of only one. Finally, using the discounted cost criterion instead of the average cost criterion will provide a further dimension to the analysis.

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